ESTIMATION OF STORE LONGITUDINAL AERODYNAMIC COEFFICIENTS THROUGH PARAMETER ESTIMATION TECHNIQUE

by

JAI KUMAR JAIN

DEPARTMENT OF AERONAUTICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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JAI KUMAR JAIN

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CERTIFICATE

4/1/88

Certified that the work presented in this thesis entitled "Estimation of Store Longitudinal Aerodynamic Coefficients through Parameter Estimation Technique" by Jai Kumar Jain has been carried out under my supervision and has not been submitted elsewhere for a degree.

S.C. Raisinghani
Professor
Department of Aeronautical Engineering
Indian Institute of Technology, Kanpur

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ABSTRACT

A formulation is proposed for the estimation of aerodynamic coefficients (parameters) of external stores through the technique of parameter estimation. The release of stores from the aircraft acts as an input and the resulting aircraft response is analysed through the Gauss-Newton method to estimate the store parameters. An example with simulated data has been used to show how the accuracy of estimation is affected by varying noise levels in the measured response, initial values of parameters used and the location of stores on the parent aircraft. Two possible approaches, called Method 1 and Method 2 are applied and compared : Method 1 obtains the aircraft as well as store parameters from a single aircraft response due to store release; Method 2 uses a two step approach - first aircraft parameters are obtained by usual input in the form of an elevator deflection and then only the store parameters are determined from the response due to store release wherein the aircraft parameters are kept fixed at a priori values obtained in step 1. Method 1 is shown to work well when measurement noise is either absent or its intensity is less while Method 2 is found to yield satisfactory results even in presence of high intensity noise in the measured data.

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LIST OF SYMBOLS

A, B	Matrix containing dimensional stability and control derivatives (parameters); Eq.2.1.
A_F	Frontal cross-sectional area of store, m^2
A_X, A_Z, A_M	Magnitude of step input following store release (Eq. 2.9a,b,c)
b	Aircraft wing span, m
$\bar{c}$	Mean aerodynamic chord of aircraft, m
ΔC	Matrix containing the increament terms of parameters (Eq. 2.12)
$C_{D_S}, C_{L_S}, C_{M_S}$	Store drag, lift and pitching moment coefficients
C_K	Parameters of the model
ΔC_K	Increament in values of C_K
$\Delta D, \Delta L, \Delta M$	Change in drag, lift and pitching moment of aircraft due to store release
D_S, L_S, M_S	Store drag, lift and pitching moment
d_S	Maximum diameter of the store, m
d_T	Vertical shift in aircraft C.G. following store release, m
g	Acceleration due to gravity, m/sec^2
I_{yy_a}	Pitch moment of inertia of aircraft, $kg\text{-}m^2$
i, j	Integers

l_t	Distance between a.c. of horizental tail and C.G. of aircraft, m
m_a	Mass of aircraft, kg.
m_S	Mass of store, kg
N	Total number of data points
n	Total number of state variables
N_S	Number of stores
P	Matrix containing partial derivatives of state variables (Eq. 2.12a)
q	Pitch rate, rad/sec
$\bar{q}$	Free stream dynamic pressure, N/m^2
q_S	Dynamic pressure seen on the store, N/m^2
S	Aircraft wing area, m^2
S_{C_F}	Scale factor = $q_S/\bar{q}$
S_S	Store wing area, m^2
t	time, sec
Δt	Time interval, sec
U	Airplane velocity, m/sec
u	Perturbation in X-component of velocity, m/sec
x	Vector containing state variables
x_m	Measured response
x_e	Estimated response
X_S	Longitudinal distance between store C.G. and C.G. of aircraft, m

Y	Vector of measurements
Y_S	Span wise location of store, m
Z_S	Vertical distance between aircraft C.G. and store C.G., m

<u>Greek Symbols</u>

α	Perturbation in angle of attack of aircraft, rad
δ_e	Elevator deflection, rad
η	Control input
θ	Perturbation in pitch angle, rad
θ_1	Steady state pitch angle, rad
$\sigma_{CR}, \bar{\sigma}_{CR}$	Cramer-Rao bounds (Eq. 3.3, 3.4)
σ_N	Standard deviation of measurement noise
σ_S	Sample standard deviation of parameter estimates
ω	Additive measurement noise
ρ	Atmospheric density, kg/m^3

<u>Subscripts</u>

1	Steady state

<u>Superscripts</u>

o	Initial value
T	Transpose of matrix or vector
$\cdot$	Derivative with respect to time
$*$	Updated values

Dimensional stability derivatives

$$X_u = \frac{-\bar{q}_1 \, S(C_{D_u} + 2\,C_{D_1})}{m_a \, U_1} \qquad (\mathrm{Sec}^{-1})$$

$$X_\alpha = \frac{-\bar{q}_1 \, S(C_{D_\alpha} - C_{L_1})}{m_a} \qquad (\mathrm{m\ sec}^{-2})$$

$$X_{\delta_e} = \frac{-\bar{q}_1 \, S \, C_{D_{\delta_e}}}{m_a} \qquad (\mathrm{m\ sec}^{-2}\ \mathrm{rad}^{-1})$$

$$Z_u = \frac{-\bar{q}_1 \, S(C_{L_u} + 2C_{L_1})}{m_a \, U_1} \qquad (\mathrm{sec}^{-1})$$

$$Z_\alpha = \frac{-\bar{q}_1 S(C_{L_\alpha} + C_{D_1})}{m_a} \qquad (\mathrm{m\ sec}^{-2})$$

$$Z_q = \frac{-\bar{q}_1 \, S \, C_{L_q} \, \bar{c}}{2m_a \, U_1} \qquad (\mathrm{m\ sec}^{-1})$$

$$Z_{\delta_e} = \frac{-\bar{q}_1 \, S \, C_{L_{\delta_e}}}{m_a} \qquad (\mathrm{m\ sec}^{-2}\ \mathrm{rad}^{-1})$$

$$M_u = \frac{\bar{q}_1 \, S\bar{c} \, (C_{m_u} + 2\,C_{m_1})}{I_{YY_a} \, U_1} \qquad (\mathrm{m}^{-1}\ \mathrm{sec}^{-1})$$

$$M_{\alpha} = \frac{\bar{q}_1 \; S\bar{c} \; C_{m_{\alpha}}}{I_{yy_a}} \qquad (\text{sec}^{-2})$$

$$M_{q} = \frac{\bar{q}_1 \; S \; \bar{c}^2 \quad C_{m_q}}{2 \; I_{yy_a} \; U_1} \qquad (\text{sec}^{-1})$$

$$M_{\delta_e} = \frac{\bar{q}_1 \; S \; \bar{c} \; C_{m_{\delta_e}}}{I_{yy_a}} \qquad (\text{sec}^{-2} \; \text{rad}^{-1})$$

where nondimensional stability derivatives used above are defined as follows[22].

$$C_{D_u} = \frac{\partial C_D}{\partial (u/U_1)} \quad ; \quad C_{L_u} = \frac{\partial C_L}{\partial (u/U_1)}$$

$$C_{L_q} = \frac{\partial C_L}{\partial (q\bar{c}/2U_1)} \quad ; \quad C_{L_{\delta_e}} = \frac{\partial C_L}{\partial \delta_e}$$

$$C_{D_{\alpha}} = \frac{\partial C_D}{\partial \alpha} \quad ; \quad C_{L_{\alpha}} = \frac{\partial C_L}{\partial \alpha}$$

$$C_{m_i} = \frac{\partial C_m}{\partial i} \; , \; i = \alpha, \; \delta_e$$

$$C_{m_q} = \frac{\partial C_m}{\partial (q\bar{c}/2U_1)}$$

CHAPTER - I

INTRODUCTION

Combat airplanes are normally equipped with a vast variety of external stores like bombs, fuel tanks, missiles and other ordnances to accomplish its operational missions. These stores are normally carried externally on wing and/or fuselage mounted pylons of carrier aircraft. No matter how potentially useful the store, if it damages or destroys the carrier or itself at release, it is of no practical value. Therefore, before any new store integration on an aircraft is cleared, it calls for detailed studies of handling qualities of the aircraft during carriage phase and also the seperation characteristics of the store following its release. Of many such studies reported in the literature, covert[1] and Schindel[2] have described the conditions for safe seperation of external stores.

The reliability of predicted seperation characteristics depends on an accurate estimation of aerodynamic coefficients (parameters) of the store in the presence of interference flow field of the carrier aircraft and other external stores present in the vicinity[3]. Information on estimation methods for aerodynamic coefficients of stores of complex shapes and in presence of complicated interference flow field effects is scarcely available in the open literature. Theoretical methods, currently available are restricted to steady state conditions of the flow field around the model. The prevailing wind tunnel methods such as captive model testing[4,5,6] and

drop model technique have been used to generate aerodynamic data for the preflight simulation studies. Wind tunnel drop model technique seems to be closer to the real situation than the captive model testing. However, both these methods are incapable of simulating the pronounced dynamic conditions of the stores encountered during release from the aircraft. Notwithstanding these limitations, a detailed survey of the methods and wind tunnel results on the aerodynamic loads associated with the external carriage of pylon-mounted stores adjacent to wing- fuselage combination has been reported by Marsden and Haines[7]. These methods for both the subsonic and supersonic speeds are reviewed in Ref. 7, giving indication of their accuracy, range of validity and extent to which they have been verified experimentally. Further, variation of loads with store position, wing geometry, Mach number is also discussed. Maddox et. al[8] have compared flight results for captive loads with the corresponding results from several wind tunnel tests as well as with the most agreeable mathematical model when conditions were matched as closely as possible. In the above study, a store similar in shape to Mk 83 bomb was mounted on a completely instrumented F-4 aircraft. The flight conditions spanned Mach 0.6-0.9 in both maneuvering and steady flight. The data showed good correlation between flight test and wind tunnel results for moderate subsonic Mach numbers but pronounced divergence as the Mach number was increased.

It is, thus, clear that a need exists to obtain aerodynamic coefficients of the captive stores in real flight conditions. To that purpose, we have proposed a method for estimating aerodynamic parameters of captive stores by employing parameter estimation technique of extracting parameters from input-output data of an aircraft. An excellent review of various methods used for parameter estimation from flight data is given by Maine and Iliff[9]. A brief outline and relative merits of various parameter estimation methods is given below.

Aircraft parameter estimation is the process of determining stability and control derivatives (Parameters) from the measured response of the aircraft for a known input. With the availability of high speed digital computers, parameter estimation has seen extensive practical applications. Following approaches are generally adopted for estimating stability and control derivatives :

- Theoretical methods
- Wind tunnel testing
- Flight testing

For the purpose of preliminary design of an airplane/store , theoretical methods are useful, inspite of their limited accuracy. The wind tunnel testing improves the accuracy of estimation but it is a time consuming and cost prohibitive process of determining parameters. Accurate

simulation of control surfaces, power effects and flight conditions is difficult. Wind tunnel models used for most of the testing are often slightly different from the actual flight vehicle because of last minute configuration changes. Reynolds number differences and presence of support system are major reasons for discrepancies between flight and wind tunnel results. For these reasons, it is always desired that the wind tunnel estimates be corroborated with the estimates of actual flight testing.

Various estimators available to extract parameters from flight data can be put under the following sub heads :

- Equation error methods
- Out put error methods
- Advanced (Statistical) methods

The equation error methods are based on the principle of least squares. It minimizes square of the error in satisfying the equations of motion with respect to unknown parameters. Each equation is solved independently. The main appeal of the method is its computational simplicity and easy application to any linear or nonlinear model. The disadvantage is that the method can not be directly applied if all states are not measured accurately and produces poor results if the measurements are noisy. Considerable effort is, therefore, required for data reconstruction and

smoothing in order to obtain accurate results. These data preprocessing tasks are sometimes more complicated than the parameter estimation using equation error. However, these methods can be very effectively used as start up methods for more advanced estimators.

The output error methods minimize the error between the measured and the model response produced for an identical input. It is assumed that the measured response is corrupted only by noise and that there are no modelling errors (process noise). Methods of this kind are capable of processing the measurement noise while assuming the model to be exact representation of the given system. These are probably the most widely used estimators for determining aircraft parameters. A comprehensive survey of these methods is reported by Maine & Iliff[10]. Newton-Raphson method[11,12] with its varients, gradient methods[9] and Analog matching method[13] fall in this category. A brief description of few of these methods is given in succeeding paragraphs.

Newton-Raphson optimization algorithm[11] is the basis for most of the second order methods (methods that use second derivatives of the cost function). It uses a two term expansion of the Taylor series. Since the evaluation of second gradient matrix is complex, the minimization of cost function is, generally very slow. Moreover, if the

second gradient is not positive definite, then the approximating function does not have a unique minimum and the algorithm is likely to behave poorly. The performance of the method in close neighbourhood of a local minimum is, still, excellent. If the initial estimates are far from the minimum, the algorithm often converges erratically or even diverges. Further, there is necessity of inverting the second gradient matrix. The crucial issue concerning the inversion is that the matrix could be singular or ill conditioned.

Modified Newton-Raphson methods[11] are used where explicit evaluation of the second gradient of the cost function is complicated or costly but the performance of the Newton-Raphson algorithm is desired. These methods approximate the second gradient in terms of the first gradient. The approximation generally improves the speed of convergence.

Advance methods are capable of determining parameters in the presence of measurement and/or process noise. These methods are based on the probabilistic concepts. Maximum a posteriori probability (MAP) estimator[9] and Maximum likelihood (ML) estimator[9,14] belong to this category. The latter estimator is one of the most favoured estimator because it yields parameter estimates that are asymptotically unbiased, consistent and efficient. When process noise is

absent and the covarience of the measurement noise is known, ML methods reduce to output error methods. In absence of measurement noise, ML methods reduce to Equation error methods. The algorithms accounting for both the measurement as well as process noise require more computer time and core. Moreover, convergence problems and other practical difficulties are often encountered.

Recently, Raisinghani and Adak[15,16] have proposed a simplified output error method, called Gauss-Newton(GN) method for aircraft parameter estimation. In this method, the model response is expanded in a Taylor series about the current trial value of the parameters and retains only the linear terms. This linearized model is substituted in the least square objective function and solved in the least square sense. It is shown that the method works satisfactorily even for the noisy data provided the initial trial values of the parameters are not too far off(upto $\pm$ 50% off true values). Since the computational effort involved is considerably less than the more advance methods like maximum likelihood method, while the accuracy obtained is fairly good, it was decided to use this method as the parameter estimator for the present work.

The problem studied here can be divided into two distinct phases : (i) to postulate a mathematical model,

(ii) to use a suitable output-error or any other estimation technique to extract aircraft and/or store parameters. The problem has been formulated by treating release of stores as the step input which excites the airplane dynamics to provide the output in the form of aircraft motion variables.

The magnitude and direction of perturbations will be a function of the airplane's dynamic characteristics, type of stores, location of stores and the release/jettisoning mechanism. The equations of motion of the aircraft for such an input have been derived and include store parameters, in addition to aircraft parameters, as unknowns. Only longitudinal parameters of the aircraft and store were estimated. It is shown that an adequate excitation of aircraft dynamics by store dropping will enable simultaneous estimation of store and aircraft parameters. However, for locations and/or type of stores leading to poor excitation of aircraft dynamics and high noise levels, two step approach is recommended : (i) estimation of aircraft (without any stores attached) parameters through control (elevator) input (ii) store parameters estimation from aircraft response due to store dropping wherein all the aircraft parameters are fixed at values estimated from the first step. This approach was also used to increase the accuracy of estimation when-ever the noise level in the measured data was high.

The problem of aircraft response following store release was formulated by Rao[17] and Raisinghani and Rao[18]. In this formulation, drag coefficient, lift coefficient and pitching moment coefficient of the store appear alone or in combination as equivalent of control derivatives in the equations of motion of the aircraft. In the present work, a more direct approach has been used to arrive at essentially the same equations of motion except for correcting a few errors and inconsistencies found between Refs. 17 and 18. It may be emphasized that the methodology proposed for store parameter estimation can be used in conjunction with any one of the methods available in the literature for aircraft parameter estimation.

Due to nonavailability of real flight data, the proposed methodology for store parameter estimation has been validated on simulated data only. An aircraft, resembling FIAT-G91 and missile similar to those reported in Ref. 7 were used for the case study. Four different locations of stores were used to simulate flight data following stores release. Simulated flight data were corrupted with pseudo noise of various intensities. Store parameters were extracted either along with the aircraft parameters or alone while fixing the aircraft parameters at values estimated seperately. Details of the effect of such one step or two step estimation process are reported for various locationsand for different

intensities of noise. For most of the cases, it is shown that there is a close agreement between the estimated and true values of the parameters and also the Cramer-Rao bounds (σ_{CR}) are low. Mean values and sample standard deviation (σ_S) of parameter estimates are also estimated and compared with Cramer-Rao bounds (σ_{CR}) for various noise levels. The agreement between σ_S and σ_{CR} is found to be good. Finally, it is shown that the two step procedure is able to estimate the store parameters quite accurately even when the initial values of the store parameters were set arbitrarily as far of as $\pm$ 500% off the true values and also in presence of high intensity noise.

The formulation of the equations of motion of aircraft following store release is given in Chapter 2. Details of the Gauss-Newton method used for parameter estimation are also given in same Chapter. Chapter 3 contains the details of the application of the method to a test case. The results and discussion are given in Chapter 4. Main conclusions of the present study are listed in Chapter 5.

CHAPTER - II

FORMULATION

The estimation of stability and control parameters from flight data postulates a mathematical model of the aircraft under test. An important question relates to the complexity of the assumed model. Although, increasing the complexity of the model (i.e., increasing the number of unknown parameters) may lead to better description of the aircraft motion, it may result in too many parameters being sought from a limited amount of data and thereby lead to reduced accuracy of estimates. It would be, therefore, our endeavour to postulate an appropriate model for the problem to be studied here.

For the present study, the perturbed motion of the aircraft following the stores release can be expected to be small. Further, we will be interested in analysing the aircraft response of a short duration immediately following the stores dropping. The airplane with external stores (called 'Loaded aircraft' here after) is assumed to be rigid and in a steady state, rectilinear, wing level flight. Thus, we assume that the perturbed motion of aircraft due to stores dropping can be represented by the usual decoupled set of equations of motion for longitudinal and lateral-directional motions. Since we would consider the case of symmetric load dropping only, the perturbed motion will be considered only in the plane of symmetry. In calm atmosphere, the longitudinal perturbed equation of motion in the stability axes system

can be written as follows

$$\dot{x}(t) = A\,x(t) + B\,\eta(t) \tag{2.1}$$

where

$$\dot{x}(t) = \begin{bmatrix} \dot{u} & \dot{\alpha} & \dot{q} & \dot{\theta} \end{bmatrix}^T$$

$$x(t) = \begin{bmatrix} u & \alpha & q & \theta \end{bmatrix}^T$$

$$\eta(t) = \begin{bmatrix} \delta_e \end{bmatrix}$$

$$A = \begin{bmatrix} X_u & X_\alpha & 0 & -g\cos\theta_1 \\ Z_u/U_1 & Z_\alpha/U_1 & 1+\frac{Z_q}{U_1} & -g\sin\theta_1/U_1 \\ M_u & M_\alpha & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} X_{\delta_e} & Z_{\delta_e}/U_1 & M_{\delta_e} & 0 \end{bmatrix}^T$$

Matrix, x contains the state variables, $\eta(t)$ is the control input, matrices A and B contain the dimensional stability and control derivatives as defined in the nomenclature.

The algebraic/transcendental equation of measurement can be written as

$$y(t) = h\left[x(t),\ \eta(t),\ t\right] + \omega(t) \tag{2.2}$$

where $y(t)$ is the vector of measurements, $h[\cdot]$ defines the relationship between the measurements, the state and the control, and $\omega(t)$ is a vector of additive measurement noise. Although all or some of the output variables may be different from the state variables, for the present study using simulated data only, we will assume for simplicity that either all the state variables are measured directly or have been obtained through a state estimation algorithm. Thus, the measurement equation is written as

$$y_i(t) = x_i(t) + \omega_i(t) \tag{2.3}$$

where $y_i(t)$ and $x_i(t)$ are the components of measurement and state vectors and $\omega_i(t)$ are the measurement noise components.

It must be pointed out that for all the output error methods, including the present one, the modelling error must also be included as measurement noise[19,20]. For real flight data, this may tend to make the noise quite coloured. However, for simplicity, we have assumed that the measurement noise components are independent, stationary Gaussian noise sequences. Again, in real data, there may be biases in the flight data, which would have to be included in Eq. (2.3) and estimated through the estimation algorithm. For simplicity again, we assume that our simulated data have no biases.

The above set of equations of motion were used in Refs. 15,18,21 to estimate aircraft parameters from flight data. A known control (elevator) input was applied to obtain the perturbed response of the aircraft and Gauss-Newton method was used to extract parameters from it. For the stores release case, the controls are assumed to be locked while the stores are dropped. Thus, the input to the aircraft is in the form of a step input due to dropping of the stores. The equations of motion corresponding to such an input were obtained by Raisinghani and Rao[18]. However, we shall obtain the same set of equations in a more straight forward manner by considering the input forces and moments acting on the airplane following the stores release. The following changes take place when stroes are released symmetrically from an aircraft.

(1) Gross weight reduction equal to the weight of the stores dropped.

(2) Centre of gravity (C.G.) of aircraft shifts, both vertically and longitudinally.

(3) Moments of inertia change.

(4) Contribution of forces and moments to the loaded aircraft due to stores ceases to act following stores release. The magnitude of these forces and moments depends upon the type of stores, location of stores, aircraft geometry, shift in C.G. and flight conditions etc.

(5) The area of the wing from where the stores are dropped becomes aerodynamically cleaner and, thus, changes forces and moments contribution of the wing.

(6) Till stores remain in the immediate vicinity of the wing after their seperation, interference effects cause variation in forces and moments acting on the wing.

Items 5 and 6 in the above list are very complex and difficult to estimate theoretically with any degree of confidence. It is hoped, however, that change in forces and moments due to these effects will be much smaller as compared to other effects listed above. Therefore, these have not been included in the formulation.

Following release of stores, there will be a shift in the C.G. location. The magnitude and direction of shift in C.G. will be a function of the type of stores and their location on the aircraft prior to release. For symmetric release of stores, C.G. of aircraft is assumed to shift only in the plane of symmetry. Since stores are mostly so located that the longitudinal C.G. locations of stores and ofthe aircraft are almost coincident, the change in the longitudinal location of C.G. following stores dropping is negligible and, therefore, not considered in the present formulation. However, the vertical shift in C.G. location following stores release is considered. It may be mentioned that the longitudinal shift in C.G. can be easily included in the formulation, if

desired for a particular configuration.

Let us consider an even number of identical stores located symmetrically under the wing of an aircraft. The steady state flight of such a configuration will be considered first. The equilibrium condition implies that the net forces along X and Z axes, and the pitching moment about Y-axis is zero. These forces and moments are due to the thrust, weight and aerodynamic forces of the aircraft and stores. Let the drag, lift and pitching moment acting at C.G. of the store be denoted by D_s, L_s and M_s respectively. The distances between the store and aircraft C.G. are shown in Fig. 1. The changes in the drag, lift and pitching moment of the aircraft due to store dropping can be written as follows :

$$\Delta D = -D_s N_s - m_s g N_s \sin \theta_1 \tag{2.4a}$$

$$\Delta L = -L_s N_s + m_s g N_s \cos \theta_1 \tag{2.4b}$$

$$\Delta M = -M_s N_s + L_s X_s N_s + D_s Z_s N_s - D_s N_s d'_T \tag{2.4c}$$

where N_s is the number of store (2 in our case), m_s is the mass of the store, X_s is the distance of store C.G. from the aircraft C.G. (positive behind), Z_s is the distance of store C.G. below the aircraft C.G. (positive downward), d_T is the vertical shift in the aircraft C.G. (positive down) (see Fig. 1). The last term on the right

hand side of Eq. (2.4c) arises due to the vertical shift of aircraft C.G. following the stores release; its occurance is explained as follows.

Prior to release of stores, the thrust and drag forces contributing to pitching moment were equal and opposite. After release of stores the net drag force is reduced by $D_s\ N_s$. Equivalently, we could say that the thrust force exceeds drag force by an amount equal to $D_s\ N_s$. Due to the vertical shift (d_T) in C.G. of aircraft, this excess thrust will contribute additional pitching moment about C.G. given by the last term of Eq. (2.4c).

The above changes in forces and moments following stores release are treated as step inputs to excite the aircraft dynamics. Thus, we can treat these inputs equivalent to a step control inputs whose magnitudes are determined as follows :

The control derivatives : X_{δ_e}, Z_{δ_e} and M_{δ_e} of Eq. (2.1) are defined as follows[22]

$$X_{\delta_e} = -\frac{\bar{q}_1 S\, C_{D_{\delta e}}}{m_a} = -\frac{1}{m_a}\frac{\partial D}{\partial \delta_e} \tag{2.5a}$$

$$Z_{\delta_e} = -\frac{\bar{q}_1 S\, C_{L_{\delta_e}}}{m_a} = -\frac{1}{m_a}\frac{\partial L}{\partial \delta_e} \tag{2.5b}$$

$$M_{\delta_e} = \frac{\bar{q}_1 S \bar{C} C_{M_{\delta_e}}}{I_{YY_a}} = \frac{1}{I_{YY_a}} \frac{\partial M}{\partial \delta_e} \tag{2.5c}$$

As defined above, the control derivatives X_{δ_e} and Z_{δ_e} represent for a unit step change in δ_e, the change in forces per unit mass of aircraft along X-axis and Z -axis respectively. Similarly, M_{δ_e} represents for a unit step change in δ_e, the change in pitching moment per unit I_{YY_a}. Using the changes in forces and pitching moment arising due to the stores release, the equivalent control derivatives for the present problem can be defined as follows.

$$X_{\delta_e} \equiv A_X = -\frac{\Delta D}{m_a} = \frac{D_S N_S + m_S g N_S \sin\theta_1}{m_a} \tag{2.6a}$$

$$Z_{\delta_e} \equiv A_Z = -\frac{\Delta L}{m_a} = \frac{L_S N_S - m_S g N_S \cos\theta_1}{m_a} \tag{2.6b}$$

$$M_{\delta_e} \equiv A_M = \frac{\Delta M}{I_{YY_a}}$$

$$= \frac{-M_S N_S + L_S X_S N_S + D_S Z_S N_S - D_S N_S d_T}{I_{YY_a}} \tag{2.6c}$$

Thus, A_X, A_Z and A_M will replace respectively X_{δ_e}, Z_{δ_e} and M_{δ_e} in Eq. (2.1) with $\eta(t)$ replaced by 1(unit step). Alternatively, the above expressions could be arrived at by following the formulation similar to that of Rao[17].

The drag, lift and pitching moment of store can be expressed in terms of nondimensional coefficients as follows

$$D_S = C_{D_S} \, q_S \, A_F \tag{2.7a}$$

$$L_S = C_{L_S} \, q_S \, S_S \tag{2.7b}$$

$$M_S = C_{M_S} \, q_S \, S_S \, C_S \tag{2.7c}$$

where q_S is dynamic pressure seen on the store and is given by

$$q_S = \bar{q}_1 \, S_{CF} \tag{2.8}$$

Here $\bar{q}_1$ is steady state dynamic pressure and S_{CF} is scale factor whose value is a function of type of store, location of store and the Mach number.

A_F is frontal cross-sectional area of the store.

S_S is exposed area of two wings of store.

and C_S is mean chord of store wing.

Substituting Eq. (2.7) in Eq. (2.6) we get

$$X_{\delta_e} \equiv A_X = \frac{(C_{D_S} \, q_S \, A_F + m_S \, g \sin \theta_1) \, N_S}{m_a} \tag{2.9a}$$

$$Z_{\delta_e} \equiv A_Z = \frac{(C_{L_S} \, q_S \, S_S - m_S \, g \cos \theta_1) \, N_S}{m_a} \tag{2.9b}$$

$$M_{\delta_e} = A_M = \frac{(-C_{M_S} C_S + C_{L_S} X_S)\, q_S\, S_S\, N_S + C_{D_S}\, q_S\, A_F\, N_S (Z_S - d_T')}{I_{YY_a}} \tag{2.9c}$$

Once the above model of the aircraft is assumed to be known, the system identification problem reduces to that of parameter estimation. As mentioned earlier, the Gauss-Newton algorithm for parameter estimation proposed by Raisinghani and Adak[15,16] has been used for the present work. A brief outline of the method is presented here for the sake of completion.

Let the dimensional stability derivatives appearing in matrix A and control derivatives in matrix B be denoted by C_k, $k=1,2, \ldots, m$ where m is the total number of unknown parameters of aircraft and store. State variables : u, α, q and θ are represented by $x_i(t)$, $i=1,2, \ldots, n$ where n is total number of state variables. Let the corresponding measured state variables be denoted by $x_{mi}(t)$. For the same input (elevator or store dropping) and using initial starting values of $C_k = C_k^o$, the estimated response of aircraft is obtained by solving Eq. (2.1). Let this be denoted by $x_i^o(t)$. The difference between the measured, $x_{mi}(t)$ and estimated, $x_i^o(t)$ responses is attributed to the difference between the values of C_k and C_k^o. The aim is to change C_k^o values such that $x_i^o(t) \longrightarrow x_{mi}(t)$. Let the values of C_k^o be changed by

ΔC_k so that $C_k^* = C_k^\circ + \Delta C_k$. The corresponding estimated response for $C_k = C_k^*$ is denoted by $x_i^*(t)$. Following Gauss-Newton minimization procedure, the model is linearized by expanding $x_i^*(t)$ in a Taylor series about the current trial values of the parameters and retaining the linear terms only, we get

$$x_i^*(t) = x_i^\circ(t) + \left[\frac{\partial x_i(t)}{\partial C_1}\right]^\circ \Delta C_1 + \left[\frac{\partial x_i(t)}{\partial C_2}\right]^\circ \Delta C_2 + \cdots$$

$$\cdots + \left[\frac{\partial x_i(t)}{\partial C_m}\right]^\circ \Delta C_m \qquad (2.10)$$

Here superscript 'o' means quantities evaluated at initial trial value.

A least square objective function is written as follows

$$\text{minimize} \quad S = \sum_{i=1}^{n} \sum_{j=0}^{N-1} \left[x_{mi}(t_j) - x_i^*(t_j)\right]^2 \qquad (2.11)$$

where N represents the data points used for analysis of each of n state variables.

Now the linearized model, Eq. (2.10) is substituted into the objective function, Eq. (2.11) and the equations are formed by setting the partial derivatives of S with respect to each of the unknown parameter equal to zero.

$$\frac{\partial S}{\partial C_k} = 0, \; k = 1, 2, \ldots, m$$

The resulting equations for the longitudinal case are as follows

$$(P^T P) \Delta C = P^T (\Delta X) \tag{2.12}$$

where superscript 'T' designates the transpose of the matrix and

$$P = \begin{bmatrix}
\frac{\partial u}{\partial C_1}(t_o) & \frac{\partial u}{\partial C_2}(t_o) & \cdots & \cdots & \cdots & \frac{\partial u}{\partial C_m}(t_o) \\
 & & \vdots & \vdots & \vdots & \\
\frac{\partial u}{\partial C_1}(t_{N-1}) & \frac{\partial u}{\partial C_2}(t_{N-1}) & \cdots & \cdots & \cdots & \frac{\partial u}{\partial C_m}(t_{N-1}) \\
\frac{\partial \alpha}{\partial C_1}(t_o) & \frac{\partial \alpha}{\partial C_2}(t_o) & \cdots & \cdots & \cdots & \frac{\partial \alpha}{\partial C_m}(t_o) \\
 & & \vdots & \vdots & \vdots & \\
\frac{\partial \alpha}{\partial C_1}(t_{N-1}) & \frac{\partial \alpha}{\partial C_2}(t_{N-1}) & \cdots & \cdots & \cdots & \frac{\partial \alpha}{\partial C_m}(t_{N-1}) \\
\frac{\partial q}{\partial C_1}(t_o) & \frac{\partial q}{\partial C_2}(t_o) & \cdots & \cdots & \cdots & \frac{\partial q}{\partial C_m}(t_o) \\
 & & \vdots & \vdots & \vdots & \\
\frac{\partial q}{\partial C_1}(t_{N-1}) & \frac{\partial q}{\partial C_2}(t_{N-1}) & \cdots & \cdots & \cdots & \frac{\partial q}{\partial C_m}(t_{N-1}) \\
\frac{\partial \theta}{\partial C_1}(t_o) & \frac{\partial \theta}{\partial C_2}(t_o) & \cdots & \cdots & \cdots & \frac{\partial \theta}{\partial C_m}(t_o) \\
 & & \vdots & \vdots & \vdots & \\
\frac{\partial \theta}{\partial C_1}(t_{N-1}) & \frac{\partial \theta}{\partial C_2}(t_{N-1}) & \cdots & \cdots & \cdots & \frac{\partial \theta}{\partial C_m}(t_{N-1})
\end{bmatrix} \tag{2.12a}$$

$$\Delta C \equiv \begin{bmatrix} \Delta C_1 & \Delta C_2 & \cdots & \cdots & \cdots & \Delta C_m \end{bmatrix}^T$$

$$\Delta X \equiv \begin{bmatrix} u_m(t_o) - u^\circ(t_o) \\ \vdots \\ u_m(t_{N-1}) - u^\circ(t_{N-1}) \\ \alpha_m(t_o) - \alpha^\circ(t_o) \\ \vdots \\ \alpha_m(t_{N-1}) - \alpha^\circ(t_{N-1}) \\ q_m(t_o) - q^\circ(t_o) \\ \vdots \\ q_m(t_{N-1}) - q^\circ(t_{N-1}) \\ \theta_m(t_o) - \theta^\circ(t_o) \\ \vdots \\ \theta_m(t_{N-1}) - \theta^\circ(t_{N-1}) \end{bmatrix}$$

The partial derivatives in matrix, P are function of time and parameters, C_k° . To solve the above set of Eqs.(2.12) for ΔC_k's, we need these partial derivatives. The following procedure is used to obtain the desired partial derivatives as a function of time.

The governing differential Eq. (2.1) is differentiated with respect to each of the parameters being estimated. First terms on the left-hand side of these equations will be of the type $(\frac{\partial}{\partial X_u})(\frac{\partial u}{\partial t})$ whose order of differentiation is interchanged. On the right hand side, the terms representing variation of input with parameters, C_k (such as $\frac{\partial \delta_e}{\partial X_u}$ etc) are set to zero as the control input is independent of the changes in parameter. Thus, we obtain the following set of 12 equations of the form similar to Eq. (2.1), i.e.

$$\dot{x}(t) = A\,x(t) + B\,\eta(t)$$

where A remains unchanged and $\eta(t) \equiv 1$ is used while vector x(t) and matrix B are replaced by the following sets to evaluate the partial derivatives occuring in vector x(t) :

$$\text{(i)}\quad x(t) = \left[\left(\frac{\partial u}{\partial X_r}\right) \quad \left(\frac{\partial \alpha}{\partial X_r}\right) \quad \left(\frac{\partial q}{\partial X_r}\right) \quad \left(\frac{\partial \theta}{\partial X_r}\right)\right]^T$$

$$B = \left[r \quad o \quad o \quad o\right]^T \tag{2.13a}$$

$$\text{(ii)}\quad x(t) = \left[\left(\frac{\partial u}{\partial Z_r}\right) \quad \left(\frac{\partial \alpha}{\partial Z_r}\right) \quad \left(\frac{\partial q}{\partial Z_r}\right) \quad \left(\frac{\partial \theta}{\partial Z_r}\right)\right]^T$$

$$B = \left[o \quad r \quad o \quad o\right]^T \tag{2.13b}$$

$$\text{(iii)}\quad x(t) = \left[\left(\frac{\partial u}{\partial M_r}\right) \quad \left(\frac{\partial \alpha}{\partial M_r}\right) \quad \left(\frac{\partial q}{\partial M_r}\right) \quad \left(\frac{\partial \theta}{\partial M_r}\right)\right]^T$$

$$B = \left[o \quad o \quad r \quad o\right]^T \tag{2.13c}$$

with $r = u, \alpha, q$ and δ_e used to obtain the required 12 sets of equations which can be solved to obtain the 48 partial derivatives and these are used to form the matrix P in Eq. (2.12).

Now the set of linear algebraic Eq. (2.12) can be solved by any appropriate technique for ΔC_k's. These are used to update the current estimates for C_k by writting

$$C_k^{I+1} = C_k^I + \Delta C_k \quad , \; I = 1, 2, \ldots, NI$$

where NI being the number of iterations.

Using the so obtained updated values of parameters, the aircraft response is calculated and matched with measured response. This process is continued until the matching between the estimated and measured flight response is within the specified margin or the number of iterations exceed the specified limit, whichever occurs earlier.

Since the partial derivatives are evaluated with respect to non-optimal parameters, C_k^I, at each iteration, the parameter improvement, ΔC_k, does not immediately lead to the optimal values of C_k's. However, if the process is convergent, the ΔC_k improvement will tend to zero such that the estimated aircraft response will tend towards the measured response and the C_k^{I+1} values will approach the optimal values as the number of iterations increases.

CHAPTER – III

APPLICATION OF THE METHOD AND CASE STUDY

3.1 TEST CASE

The method formulated in previous chapter was applied on a test case. An aircraft resembling FIAT G-91 was selected. Its inertia and geometric characteristics and stability and control derivatives[13] are listed in Table 1. The store was selected from Ref. 7 so that its aerodynamic coefficients could be estimated using the graphs given in that reference. The geometric details of the store are also given in Fig. 1.

Since, no real flight data could be obtained for the aircraft response following stores release, the method was applied on the simulated data. For this purpose, a computer program in Fortran IV was developed. A flow chart of the computer program used for parameter estimation is shown in Fig. 2. The main features of the program are as follows :

(i) For the true values of the parameters and known input form, it generates simulated measured response of the aircraft. The input can be given either in the form of an arbitrary elevator input (Table 2 and Fig. 3) or in the form of stores release. This kind of response will be referred to as no-noise response.

(ii) Measurement noise of various intensities could be added to the no-noise response to generate noisy responses. Pseudo random numbers were computer generated so as to have

a normal distribution with zero mean and assigned standard deviation. The intensity of noise was varied to correspond approximately to 1%, 2%, 5% and 10% of the maximum magnitude of the corresponding motion variable. A typical set of standard deviation values (σ_N) of noise used for all the four store locations corresponding to 5% noise are shown in Table 3.

(iii) Using the initial values of the parameters assigned for the model, it generates model response by solving Eq. (2.1) for the same input as was used to generate the measured response. The input could be an elevator deflection or a step input due to store release.

(iv) It calculates the partial derivatives (sensitivity coefficients) as required to form matrix, P of Eq. (2.12) by solving Eq. (2.13).

(v) The set of Eqs. (2.12) is arranged and solved to obtain the increaments, $\Delta C_k^{\prime}s$, for the parameters being estimated and the values of the parameters are updated. It has the option of utilizing either no-noise response or noisy response obtained in steps (i) and (ii) above.

(vi) Using increamented parameters as initial values, steps (iii) to (v) are repeated each time, till the model response and measured response match within the specified accuracy

or the change in successive values of the parameters is less than the specified or the number of iteration exceeds the assigned limit.

(vii) There is an option for using the above basic scheme in the following two ways.

(a) To estimate all the aircraft as well as store parameters from the measured response due to stores dropping in one step. This will be referred to as Method 1.

(b) The parameters of the aircraft can be first estimated by analysing aircraft response to known elevator input. Next, these values of aircraft parameters are kept fixed at estimated values and the estimation algorithm used again to estimate only the store parameters from the aircraft response due to stores dropping. This we shall refer to as method 2.

For solving Eq. (2.1) to obtain measured and model response, a fourth order Runge-Kutta method was employed. A step size of 0.005 second was used and the data was scrambled back to a step size of 0.1 second for the purpose of GN method. This procedure was also followed for obtaining sensitivity coefficients at step size of 0.1 second. A signal length of 4.9 seconds (50 data points) was generated.

The pseudo numbers for noise contamination were generated by a built-in computer subroutine, GGNOR. The noisy measured

responses with 1% to 10% noise levels were analysed by using both Method 1 and Method 2. The noise samples for all the motion variables were independent of each other. Further, for studying the effect of noise on parameter estimation, 20 samples of noise with same mean and standard deviation were generated and added to the same no-noise response to prepare twenty samples of measured noisy responses. These twenty responses yielded twenty different estimates of parameters which were utilized to obtain statistical properties of the estimates. Specifically, the sample mean and sample standard deviation of parameter estimates was obtained as follows

$$\bar{C} = \frac{1}{N} \sum_{i=1}^{N} C_i \tag{3.1}$$

$$\sigma_S = \left[\frac{1}{N-1} \sum_{i=1}^{N} (C_i - \bar{C})^2 \right]^{\frac{1}{2}} \tag{3.2}$$

where

$\bar{C}$ = Mean of parameter estimates

C_i = Parameter estimate of ith sample

N = Number of parameter estimates (20 in present study)

σ_S = Sample standard deviation

The computer program also contains provision for obtaining Cramer-Rao (CR) bounds for each of the estimated parameter. The CR bound[12] provides the minimum variance for parameter with which it can be estimated for a given set of data.

Thus, it provides the amount of confidence to be placed in the various parameter estimates. Since we are dealing with simulated data where the standard deviation of noise (σN) is known, the CR bounds could be obtained in the following two ways.

(i) Using the magnitude of the difference between the measured and estimated response, $x_{m_i} - x_{e_i}$ the CR bound can be estimated as follows.

$$\left[\sigma^2_{CR}\right] = \left[P^T D\, P\right]^{-1} \tag{3.3}$$

where

$$\sigma^2_{CR} = \begin{bmatrix} \sigma^2_{CR_1} & - & - & \cdots & - \\ - & \sigma^2_{CR_2} & - & \cdots & - \\ & & & \vdots & \\ - & - & - & \cdots & \sigma^2_{CR_m} \end{bmatrix}$$

$$D = \left[\begin{array}{c|c|c|c} D_{u/U_1} & 0 & 0 & 0 \\ \hline 0 & D_{\alpha} & 0 & 0 \\ \hline 0 & 0 & D_q & 0 \\ \hline 0 & 0 & 0 & D_{\theta} \end{array}\right]$$

Where D_x is a diagonal matrix of size N x N with diagonal elements (d_{ii}) given by

$$d_{ii} = \frac{1}{\sum_{i=1}^{N} \left[x_m(t_i) - x_e(t_i)\right]^2 / N} \quad ; \; i = 1, N$$

and $x = u/U_1, \alpha, q, \theta$

(ii) Using the known σ_N for all the measured variables, the lower CR bounds in terms of variance for each of the parameter estimates is given by

$$\left[\bar{\sigma}^2_{CR}\right] = \left[P^T \bar{D} \; P\right]^{-1} \tag{3.4}$$

with

$$\left[\bar{\sigma}^2_{CR}\right] = \begin{bmatrix} \bar{\sigma}^2_{CR_1} & - & - & \cdots & - \\ - & \bar{\sigma}^2_{CR_2} & - & \cdots & - \\ - & - & \bar{\sigma}^2_{CR_3} & \cdots & - \\ & & & \vdots & \\ - & - & - & \cdots & \bar{\sigma}^2_{CR_m} \end{bmatrix}$$

$$\bar{D} = \left[\begin{array}{c|c|c|c} \bar{D}_{u/U_1} & 0 & 0 & 0 \\ \hline 0 & \bar{D}_{\alpha} & 0 & 0 \\ \hline 0 & 0 & \bar{D}_q & 0 \\ \hline 0 & 0 & 0 & \bar{D}_{\theta} \end{array}\right]$$

$$\bar{D}_x = \begin{bmatrix} 1/\sigma^2_{N_x} & 0 & 0 \cdots\cdots 0 \\ 0 & 1/\sigma^2_{N_x} & 0 \cdots\cdots 0 \\ & & \vdots \\ 0 & 0 & 0 \cdots\cdots 1/\sigma^2_{N_x} \end{bmatrix}$$

where $x = u/U_1, \alpha,$ q and θ

Matrix P is given by Eq. (2.12a)

Thus, it was possible to compare σ_S with CR bounds, σ_{CR} and $\bar{\sigma}_{CR}$ obtained by using Eqs. (3.3) and (3.4).

3.2 NUMERICAL EXAMPLE

To validate the computer code, measured response of the aircraft following stores dropping was obtained for the various store locations given in Table 5. The parameters for

for the store(Fig.1) were estimated from Ref. 7 for the purpose of present study. The true values of store parameters: C_{D_S}, C_{L_S} and C_{M_S} so obtained (Table 5) were used to calculate A_X, A_Z and A_M as required to solve Eq. (2.1). Four store locations, named V1C1, V1C2, V2C1 and V2C2 were chosen. Notation V1 & V2 refer to store C.G. lying vertically below aircraft C.G. by a distance of 0.15 $\bar{C}$ and 0.4 $\bar{C}$ respectively. While C1 and C2 refer to store C.G. lying longitudinally away from aircraft C.G. by a distance of - 0.25$\bar{C}$ and 0.25 $\bar{C}$ respectively. Store C.G. lying below and behind aircraft C.G. is taken positive. The spanwise location for all the cases was fixed at half semi span position of the aircraft wing.

To illustrate, details of numerical calculations employed for obtaining A_X, A_Z and A_M for one particular location V2C2 are given below. This location means (refer to Fig. 1)

$$X_S = 0.25\ \bar{C} = 0.5125 \text{ m}$$

$$Y_S = 0.5(b/2) = 2.15 \text{ m}$$

$$Z_S = 0.4\ \bar{C} = 0.82 \text{ m}$$

Store drag coefficient, C_{D_S} is assumed to be constant and equal to 0.2 based on the frontal cross-sectional area, A_F. The dynamic pressure (q_S) seen on the store is calculated from $q_S = \bar{q}_1 S_{CF}$, where S_{CF} is obtained from Fig. 5 of Ref. 7. N_S is the number of stores dropped. For the store under

study we have

$m_S = 50$ kg ; $A_F = \frac{\pi}{4} d_S^2 = 0.0314\ m^2$

$c_S = 0.24$ m ; $S_S = 0.23\ m^2$

$X_{CGS} = 0.25\ c_S = 0.06$m ; $N_S = 2$

$\theta_1 = 0.0$ rad ; $m_a = 5000$ kg (refer Table 1)

$\rho = 0.685$ kg/m^3 ; $S_{CF} = 0.915$

$U_1 = 137.5$ m/sec ; $I_{YY_a} = 30400$ kg-m^2

$$q_S = \bar{q}_1\ S_{CF} = 0.5\rho\ U_1^2\ S_{CF} = 5925.0\ N/m^2$$

$$A_X = \frac{(C_{D_S}\ q_S\ A_F + m_S\ g\ \sin\theta_1)\ N_S}{m_a} = 0.01488\ m/sec^2$$

The values of the store lift coefficient, C_{LS} (based on the exposed area (S_S) of two wings of the store) and the store pitching moment, C_{M_S} (based on the mean chord (c_S) of the store) obtained from Fig. 11 of Ref. 7 are :

$C_{L_S} = 0.7$; $C_{M_S} = -0.2$

Now, A_Z is calculated from the following equation

$$A_Z = \frac{(C_{L_S}\ q_S\ S_S - m_S\ g\ \cos\theta_1)\ N_S}{m_a} = 0.1854\ m/sec^2$$

Upward shift in C.G. location of the aircraft following stores release is given by

$$d_T = \frac{- Z_S\ m_S\ N_S}{m_a} = - 0.0164 \text{ m}$$

The value of A_M can now be calculated from the following equation.

$$AM = \frac{(-C_{M_S}\ C_S + C_{L_S}\ X_S)\ q_S\ S_S\ N_S + C_{D_S}\ q_S\ A_F\ N_S (Z_S - d_T)}{I_{YY\ a}}$$

$$= 0.03851 \text{ sec}^{-2}$$

CHAPTER - IV
RESULTS AND DISCUSSIONS

4.1 GENERAL

In this chapter, we shall discuss the results of the test case for estimation of store parameters. The response of test airplane following stores release is utilized to estimate desired parameters through GN method. The airplane and the stores used for the study, the governing equations of motion and GN method were described in earlier chapters. The proposed method has been used to extract longitudinal store parameters from no-noise response as well as from noisy response of aircraft following stores release. Both Method 1 and Method 2 described in Chapter 3 have been extensively applied on simulated noisy response and relative merits of these methods are pointed out. Attention was also focussed on the effect of noise level present in the measured response and the effect of initial values of parameters on the accuracy of the estimated parameters. To this purpose, the results obtained for noisy responses will be discussed under the following subheads. All the results using Method 1 are discussed under subhead [redacted] of case 1 whereas the results using Method 2 are further divided under subheads [redacted] of case 2 to case 5. These cases differ from each other in the way the two steps of method 2 are used, i.e., the difference in the approach used for the estimation of aircraft parameters in step one and/or the estimation of store parameters in step 2. For ready reference, approach used for cases 2 to 5 are defined below :

- Case 2 One measured noisy response for an arbitrary elevator input was used for aircraft parameters estimation. The aircraft parameters were kept fixed at these estimated values while estimating store parameters from aircraft response due to stores release.

- Case 3 Step 1 was again similar to case 2, i.e., aircraft parameters were estimated from one noisy response for an arbitrary elevator input. The aircraft parameters were kept fixed at these estimated values while estimating the store parameters from 20 different noisy responses due to stores release.

- Case 4 The aircraft parameters were estimated from 20 different samples of noisy responses for an arbitrary elevator input. The mean values of the aircraft parameters so estimated were used as the initial values and kept fixed while estimating the store parameters from one noisy response due to stores release.

- Case 5 In this case, step one of Method 2 was replaced by choosing true values of aircraft parameters and keeping them fixed while estimating the store parameters from one noisy response due to stores release.

The Cramer-Rao bounds (σ_{CR}) for the estimated parameters were also obtained for cases 1,2,4 and 5. The mean and the sample standard deviation (σ_S) of parameter estimates for store parameters were obtained for case 3. A comparison of σ_S and σ_{CR} is also presented.

The initial values of the aircraft parameters were arbitraily chosen to be $\pm$50% off the true values while the store parameters were $\pm$500% off the true values. Many such combinations were tested to validate the computer code; however, results will be presented for one such set of initial values shown in Table 4 and 5 : both the true values and the initial values of store parameters at different locations of the stores are shown in Table 5. For convenience, the response of the model with the estimated values of the parameters will be referred to as the 'estimated response' while that with the initial values of the parameters will be referred to as the 'initial response'.

4.2 RESULTS FOR NO-NOISE CASE

Aircraft and store parameters were estimated from no-noise measured response for all the four store locations. In all cases, parameter estimates converged very close to their true values in about five iterations. Since the estimated values and true values of all the aircraft as well as store

parameters matched up to four decimal places, no comparison of these values is shown in a tabular form. However, a comparison of the measured and estimated response for all the four measured variables : u, α, q and θ for one typical location V1C1 is shown in Fig. 4. Initial response is also shown on the same figure for comparison. It is seen that the matching between the measured response and the estimated response is almost perfect. Thus, we may conclude that the proposed method can extract all the aircraft as well as store parameters quite accurately through GN method (Via Method 1) from no-noise response of aircraft due to stores release.

4.3 RESULTS FOR NOISY CASES

To study the effect of measurement noise on the accuracy of parameter estimation, noisy responses contaminated with 1%, 2%, 5% and 10% noise were analysed. For convenience, results from noisy responses are presented seperately for Method 1 and Method 2. A comparison between results for various cases will also be presented, where appropriate.

4.3.1 Results for Method 1 and Case 1

Case 1 was studied for all store locations and for noise levels of 1%, 2% and 5%. The estimated values of the parameters and their CR bounds, both σ_{CR} and $\bar{\sigma}_{CR}$ given by Eqs. (3.3) and

(3.4) for location V2C2 are shown in Table 6a,b. Table 6a shows that for low noise levels (up to 2%), all the parameters are well estimated except for the so called weak derivative, Z_q[15,21]. Notwithstanding poor estimation of Z_q, the estimated and measured responses matched quite closely as shown in Fig. 5 for one typical location V2C2 and for noise level of 2%. It may be noted from Table 6a that the true values lie within $\pm \sigma_{CR}$ of the estimated values. Further, σ CR are quite small for low noise levels and, as expected, increases proportionately for higher noise levels. However, the estimated values of the store parameters C_{D_S}, C_{L_S} and C_{M_S} for 5% noise do not compare well with the true values and have relatively large σ_{CR}. Better results were obtained by use of Method 2 as discussed in subsequent subsections.

Cramer-Rao bounds defined as σ_{CR} and $\bar{\sigma}_{CR}$ for all noise levels were also compared as shown in Table 6b. As expected, for the simulated data, the values of σ_{CR} and $\bar{\sigma}_{CR}$ are quite close to each other. It was further desired that both σ_{CR} and $\bar{\sigma}_{CR}$ be compared with sample standard deviation (σ_S) of the parameter estimates. For this purpose, one typical response data was contaminated with 20 different samples of noise having the same mean and standard deviation (σ_N). Analysing these 20 samples, the mean values of the parameter estimates and σ_S were calculated using Eqs. (3.1) and (3.2) respectively.

Values so obtained are given in Table 7 for noise levels 1%, 2% and 5%. It is noted that the mean values of all the parameters, particularly of the store parameters show considerable improvement. Further, the sample standard deviation also are smaller.

A comparison among σ_S, σ_{CR} and $\bar{\sigma}_{CR}$ for the estimated parameters for different noise levels is shown in Table 8. It shows that the ratios $\frac{\sigma_S}{\sigma_{CR}}$ and $\frac{\sigma_S}{\bar{\sigma}_{CR}}$ lie in the range of 0.75-1.3 which is considered to be a reasonably good agreement[20] for a statistical sample of 20 only.

The above results of Case 1 show that the store parameters estimation is not so accurate for high noise levels. It was conjectured that this was due to an attempt to estimate too many parameters from insufficient information contained in the measured response being analysed. This motivated the introduction of Method 2 for which the results are presented next.

4.3.2 <u>Application of Method 2</u>

To improve the accuracy of estimated store parameters, a two step approach, called Method 2 and explained in Chapter 3, was adopted. Airplane without stores is excited by an arbitrary elevator input (Table 2 and Fig. 3) to obtain airplane response, which is used to obtain airplane parameters through GN method.

The aircraft parameters and CR bounds so obtained for no-noise case and for noise levels of 1% to 10% are presented in Tables 9a and 9b. These are the values of aircraft parameters that were used as a priori fixed values in Case 2 and Case 3 when step 2 was employed to estimate store parameters. Also, twenty different noisy responses for an identical elevator input were generated by adding 20 different noise samples of same mean and standard deviation (σ_N). These were also analysed through GN method to obtain mean values for aircraft parameters. These values are presented in Table 10 and were used as a priori fixed values in Case 4. From Tables 9 and 10, it may be observed that the so called weak parameter, Z_q is again poorly estimated, as was also observed earlier for Method 1. Further, another weak parameter, X_{δ_e} shows relatively poor accuracy of estimation. However, it may be pointed out that the value of X_{δ_e} is not required for step 2 of Method 2. It may be pointed out that another weak derivative Z_{δ_e} was estimated through M_{δ_e} by relating Z_{δ_e} and M_{δ_e} by means of the known geometry of the aircraft as given below:

$$Z_{\delta_e} = \frac{M_{\delta_e} \; I_{yy_a}}{l_t \; m_a}$$

The weak parameters X_{δ_e} and Z_q are so called, since they do not affect the response significantly. That is why, inspite of poor estimates for X_{δ_e} and Z_q, the matching between the measured and estimated response was found to

be good for all the motion variables. One such typical matching for noise level of 5% is shown in Fig. 6. The results for all the four variables u, α, q and θ show that the estimated response approaches the noisy measured response such that the random fluctuations of flight data due to presence of noise are quite evenly distributed on either sides of the estimated response.

4.3.2.1 Results for Case 2

As mentioned earlier, the aircraft parameters from Table 9 were used as a priori fixed values while store parameters were estimated from the noisy response due to stores release. Following this procedure, store parameters were estimated for all four locations and for noise levels 1%, 2% and 5%. A comparison of these results with that of Case 1 for corresponding store location and noise level is given in Tables 11 through 14. It is seen that the accuracy of parameter estimates in Case 2 has improved substantially, especially at higher noise levels (see results for 5% noise level for all locations). However, at low noise levels, there is little to choose between results for Case 1 or Case 2, although the CR bounds for Case 2 are smaller as compared to those for Case 1. Further, in Case 2, fewer (2 or 3) iterations were required for convergence, making it computatio-nally economical. Matching of noisy measured response and

estimated response for one typical store location V2C2 and noise level of 5% is shown in Fig. 7. The estimated response for all the motion variables looks to be matching well with the measured response except for fluctuations in the measured response due to noise. This observation is similar to that mentioned in §4.3.2. THe estimated response seems to approach the average smoothed response one would expect from the corresponding no-noise measured response.

4.3.2.2 Results for Case 3

As in Case 2, here again aircraft parameters were kept fixed at values given in Table 9. Twenty measured responses with 20 different noise samples were analysed. Although results were obtained for all the four store locations, the results are presented for one typical location V1C2 and for noise levels varied from 1% to 5%. Mean values and σ_S for all the store parameters were obtained using Eqs. (3.1) and (3.2). The mean values and σ_S of store parameters are shown in Table 15./This comparison is also shown in Table 15. A close look at the results for these two cases shows that the results from case 2 are marginally better than Case 3 for low noise levels (up to 2%) whereas for

/ It is of interest to compare the results for case 2 and case 3.

higher noise levels, case 3 results were definitely superior to case 2. This can be explained easily based on the fact that the results for case 2 will be different depending on the chosen sample of noise to generate the measured responses It is, therefore, the results of case 3 which are representative of all such possible results for case 2, in the sense of giving mean of all possible estimates from case 2. Thus, the comparison of Case 2 and Case 3 is of only academic interest. It may be emphasized again that the improvement obtained for Case 2 or Case 3 as compared to Case 1, is the most relevant fact to be observed because of its importance to any practical application of the proposed scheme. Ratio $\frac{\sigma_S}{\sigma_{CR}}$ lie between 0.74 and 1.2 indicating that estimated CR bounds are quite comparable to σ_S as was mentioned earlier for case 2 also.

4.3.2.3. Results for Case 4

In this case, the aircraft parameters were fixed at values given in Table 10. The estimates for store parameters from noisy response due to store release are presented in Table 16 for noise levels of 1%, 5% and 10% for a typical store location of V2C1. The results for the above noise levels are also compared with those obtained for Case 2 in Table 16. It may be noticed that for all noise levels, the results for Case 4 are better than those for Case 2,

except for the parameter C_{D_S} whose accuracy deteriorates as noise level increases. However, even at 10% noise level, the matching between the estimated response and the measured response was good as shown in Fig. 8. As mentioned for Case 2 (Fig. 7), the estimated response again matches with what one would obtain from measured response after filtering out the noise.

4.3.2.4 Results for Case 5

In this case, aircraft parameters were fixed at their true values and only store parameters were estimated from noisy response due to stores release. Store parameters estimation was made for all the four store locations and noise was varied from 1% to 10%. A comparison among the results of case 2,4 and 5 is made for typical store location V2C1. Estimated values and CR bounds of all store parameters are given in Table 16. This table shows that the results obtained for Case 5 and Case 4 are almost identical, both in the estimated values as well as in their CR bounds. Thus, the observations made about the comparison between Case 4 and Case 2 results also applies to comparison of results for Case 5 and Case 2. This case is of academic interest in the sense, that it can not be used in practice since the true values of the aircraft parameters can never be postulated.

However, it may be mentioned that Case 5 represents the theoretical limit for Case 4 as far as expected accuracy of estimation is concerned. It also suggests the obvious that one should use the best possible estimates of aircraft parameter to analyse the noisy response due to stores release for obtaining the store parameters.

CHAPTER - V

CONCLUSIONS

A method has been proposed for estimation of store parameters from measured response due to stores release. The Gauss-Newton method for parameter estimation was applied to a test case to show the applicability of the proposed scheme on simulated data. Results were obtained for varying noise levels in measured data, for different store locations and initial values of parameters. The effect of using Method 1 and Method 2 on the accuracy of estimated parameters has also been presented. Based on the results presented, we can draw the following important conclusions :

(i) Application of the proposed scheme to no-noise response due to stores release can estimate all parameters of aircraft as well as store in a single step following Method 1. Further, accurate estimates with low CR bounds could also be obtained from the noisy response through method 1, if the noise levels were low ($\simeq$1%).

(ii) For noisy response with high noise levels, Method 2 is recommended. The accuracy of store parameters can be increased by using the best available estimates of the aircraft parameters. To this purpose, many methods and techniques have been reported in the literature[10] whereby one can improve the accuracy of the aircraft parameter estimates-these include design of proper flight maneuvers, choice of optimal input forms, instrumentation and

recording systems and processing of data. Also, it must be mentioned that the proposed scheme for store parameter estimation can be used in conjunction with any one of the parameter estimation techniques (e.g. maximum likelihood, filter error methods, equation error methods, regression methods etc.) available in the literature.

(iii) For a given aircraft, the type of store and location will also influence the accuracy of store parameters estimates. Also, depending on the dynamic characteristics of the aircraft, a store release will dictate the resulting response of the aircraft following its release. Thus, any combination of aircraft and store that results in better dynamic response of the aircraft following stores release will yield relatively accurate estimates of store parameters.

(iv) A comparison of sample standard deviation (σ_S) of parameter estimates with Cramer-Rao bounds (σ_{CR}) obtained in two different ways was carried out. Both σ_S and σ_{CR} were shown to be in reasonable agreement with each other. Thus, Cramer-Rao bounds are good practical measure for the sample standard deviation of parameter estimates and will provide a satisfactory indication of the reliability or confidence level of the estimated parameters.

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******************************
FILE:STR11.FOR
******************************
THIS PROGRAM ESTIMATES PARAMETERS OF AIRCRAFT AND STORE
IN SINGLE STEP.THIS HAS BEEN REFERRED TO AS METHOD 1.
MEANING OF THE VARIABLES NOT EXPLAINED HERE ARE GIVEN
IN FILE:RSP.FOR THROUGH COMMENT STATEMENTS.
MS=MASS OF TWO STORES;NS=NUMBER OF STORES.
REAL IYYA,MA,MS,NS
REAL MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE,MDEL1
DIMENSION PD(4,12,50),U(981),ALPHA(981)
DIMENSION Q(981),THETA(981),A(200,11),B(200,1),X(11,1)
DIMENSION U21(981),ALFA21(981),Q21(981),TETA21(981),WKAREA(200)
DIMENSION SIG(4),R1(49),R2(49),R3(49),R4(49)
DIMENSION DD(11,20),DDM(11),STDVN(11),SUM(11),SMTN(11)
DIMENSION D(200,200),AT(11,200),E(11,200),F(11,11)
DIMENSION UNIT(11,11),WKSPCE(13),DEF(11),CRB(11)
UM,ALPHAM,QM,THETAM ARE MEASURED MOTION VARIABLES.
UE,ALPHAE,QE,THETAE ARE ESTIMATED MOTION VARIABLES.
DIMENSION UM(50),ALPHAM(50),QM(50),THETAM(50)
DIMENSION UE(50),ALPHAE(50),QE(50),THETAE(50)
EXTERNAL UDOT,ALPDOT,QDOT
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
OPEN(UNIT=1,DEVICE='DSK',FILE='RSP.DAT',)
OPEN(UNIT=2,DEVICE='DSK',FILE='TRSP.OUT')
OPEN(UNIT=3,DEVICE='DSK',FILE='ERSP.OUT',)
OPEN(UNIT=6,DEVICE='DSK',FILE='MRSP.OUT')
OPEN(UNIT=21,DEVICE='DSK',FILE='TPRSP.OUT')
OPEN(UNIT=10,DEVICE='DSK',FILE='RSP.OUT')
TOTTIM=TOTAL TIME; TIMINC=TIME INCREAMENT
U1=STEADY STATE SPEED
KCOUNT=1 GIVES TRUE RESPONSE OF A/C FOLLOWING STORES RELEASE
KCOUNT=2 IS FOR NOISY(MEASURED) AND ESTIMATED(MODEL)RESPONSE
KCOUNT=3 IS USED FOR ESTIMATION OF PARTIAL DDERIVATIVES AND
PARAMETERS OF A/C AND STORE.
II=1
ITER=1
SCF=SCALE FACTOR;VHS=VERTICAL DISTANCE BETWEEN A/C C.G.
AND C.G. OF STORE(M);CDS,CLS,CMS ARE DRAG,LIFT AND PITCHING
MOMENT COEFFICIENTS(PARAMETERS)OF STORE RESPECTIVELY.
SS=EXPOSED AREA OF TWO WINGS OF STORE(M**2);CS=MEAN CHORD
OF STORE WING(M);DS=MAXIMUM DIAMETER OF STORE(M);XS=DISTANCE
XCG=FROM L.E. OF M.A.C. OF AIRCRAFT WING AND C.G. OF STORE(M)
RHO=AIR DENSITY(KG/M**3)
READ(1,*)TOTTIM,TIMINC,U1,KCOUNT,NOISE,INMAX,ITRMAX
READ(1,*)THETA1,RHO,G,SCF
READ(1,*)MA,VHS,S,CD1,IYYA,XCG
IF((KCOUNT.EQ.1).OR.(NOISE.EQ.0))GO TO 310
READ(1,*)IPERS,SIG(1),SIG(2),SIG(3),SIG(4),ISEED
READ(1,*)XU,XTU,XALPHA,XQ,XDELE
READ(1,*)ZU,ZALPHA,ZALDOT,ZQ,ZDELE
READ(1,*)MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
READ(1,*)MS,CDS,CLS,CMS,SS,CS,DS,NS,XS
IF(IN.EQ.1)GO TO 8
GO TO 10
ISEED=ISEED+I*557255
GO TO 6
-------------------------------------------------------------------------------
-----WRITE INPUT VALUES-----
WRITE(10,11)
FORMAT(//1X,'INPUT:'/4X,'STARTING INPUT PARAMETER VALUES')
WRITE(10,12)XU,XTU,XALPHA,XQ,XDELE
```

```
FORMAT(/6X,'XU=',E14.6/6X,'XTU=',E14.6/6X,'XALPHA=',E14.6/
1   6X,'XQ=',E14.6/6X,'XDELE=',E14.6)
WRITE(10,13)ZU,ZALPHA,ZALDOT,ZQ,ZDELE
FORMAT(/6X,'ZU=',E14.6/6X,'ZALPHA=',E14.6/6X,'ZALDOT=',E14.6/
1   6X,'ZQ=',E14.6/6X,'ZDELE=',E14.6)
WRITE(10,14)MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
FORMAT(/6X,'MU=',E14.6/6X,'MTU=',E14.6/6X,'MALPHA=',E14.6/
1   6X,'MTALPA=',E14.6/6X,'MALDOT=',E14.6/6X,'MQ=',E14.6/
1   6X,'MDELE=',E14.6)
WRITE(10,15)TOTTIM,TIMINC,U1,KCOUNT,NOISE,INMAX,ITRMAX
FORMAT(/6X,'TOTAL TIME=',E14.6/
1   6X,'TIME INCREAMENT=',E14.6/6X,'U1=',E14.6,5X,
1   'KCOUNT=',I2,5X,'NOISE=',I3,5X,'INMAX=',I3/5X,'ITRMAX=',I3)
WRITE(10,20)MA,VHS,S,CD1,IYYA,XCG
FORMAT(3X,'MA=',F10.2,3X,'VHS=',F7.4,3X,'S=',F7.2,3X,'CD1=',
1  F7.4 ,3X,'IYYA=',F10.2,3X,'XCG=',F7.4)
WRITE(10,21)MS,CDS,CLS,CMS,SS,CS,DS,NS,XS
FORMAT(3X,'MS=',F6.1,3X,'CDS=',F7.4,3X,'CLS=',F7.4,3X,'CMS=',
1 F8.4,3X,'SS=',F5.2/3X,'CS=',F5.2,3X,'DS=',F5.2,3X,'NS=',F4.1,
1    3X,'XS=',F7.4)
WRITE(10,22)THETA1,RHO,G,SCF
FORMAT(3X,'THETA1=',F5.2,3X,'RHO=',F6.3,3X,'G=',F5.2,
1   3X,'SCF=',F6.3)
IF((KCOUNT.EQ.1).OR.(NOISE.EQ.0))GO TO 6
WRITE(10,16)PERN,SIG(1),SIG(2),SIG(3),SIG(4),ISEED
FORMAT(5X,'PERN=',F5.2,5X,'SIG(1)=',F10.6,5X,'SIG(2)=',F10.6/
1    5X,'SIG(3)=',F10.6,5X,'SIG(4)=',F10.6,5X,'ISEED=',I12)
------------------------------------------------------------------
CONTINUE
-----INTIAL FLIGHT CONDITIONS ------
DATA THETA(1)/0.0/,U(1)/0.0/,Q(1)/0.0/
DATA ALPHA(1)/0.0/,PI/3.1416/
------------------------------------------------------------------
KTOT=981
IP=0.1/TIMINC
AF=FRONTAL CROSS-SECTIONAL AREA OF STORE;QB=FREE STREAM DYNAMIC
PRESSURE(N/M**2);QS=DYNAMIC PRESSURE ON STORE(N/M**2)
DT=VERTICAL SHIFT IN C.G.OF A/C FOLLOWING STORES RELEASE(M).
AF=3.1416*DS*DS/4.0
QB=0.5*RHO*U1*U1
QS=SCF*QB
DT=-VHS*(MS/MA)
------ESTIMATION OF STEP INPUTS FOLLOWING STORES RELEASE-------
AX=(CDS*QS*AF*NS+MS*G*SIN(THETA1))/MA
AZ=(CLS*QS*SS*NS-MS*G*COS(THETA1))/MA
AM=((-CMS*CS-(XCG-XS)*CLS)*QS*SS*NS+CDS*QS*AF*NS*(VHS-DT))/IYYA
WRITE(10,19)AX,AZ,AM
FORMAT(5X,'AX=',F8.5,5X,'AZ=',F8.5,5X,'AM=',F8.5)
 XDELF=AX
 ZDELF=AZ
 MDELF=AM
 GO TO 31
 REWIND(21)
 READ(21,*)(U21(K),ALFA21(K),Q21(K),TETA21(K),K=1,KTOT)
 JCOUNT=12
 IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))JCOUNT=1
 DO 50 J=1,JCOUNT
 T=0.0
 IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))GO TO 35
 -----ESTIMATION OF PARTIAL DERIVATIVES -----
 E1=QS*AF*NS
 E2=QS*SS*NS
 IF((J.EQ.1).OR.(J.EQ.2).OR.(J.EQ.3))GO TO 610
 IF(J.EQ.4)GO TO 611
 IF((J.EQ.5).OR.(J.EQ.6).OR.(J.EQ.7))GO TO 620
 IF(J.EQ.8)GO TO 621
```

```
IF(J.EQ.12)GO TO 622
XDELE=0.0
ZDELE=0.0
MDELE=1.0
GO TO35
XDELE=1.0
ZDELE=0.0
MDELE=0.0
GO TO 35
XDELE=E1/MA
ZDELE=0.0
MDELE=E1*(VHS-DT)/IYYA
GO TO 35
XDELE=0.0
ZDELE=1.0
MDELE=0.0
GO TO 35
XDELE=0.0
ZDELE=E2/MA
MDELE=-(XCG-XS)*E2/IYYA
GO TO 35
XDELE=0.0
ZDELE=0.0
MDELE=-E2*CS/IYYA
DO 50 K=1,KTOT
IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))GO TO 700
IF((J.EQ.1).OR.(J.EQ.5).OR.(J.EQ.9))DELE=U21(K)
IF((J.EQ.2).OR.(J.EQ.6).OR.(J.EQ.10))DELE=ALFA21(K)
IF((J.EQ.3).OR.(J.EQ.7).OR.(J.EQ.11))DELE=Q21(K)
IF((J.EQ.4).OR.(J.EQ.8).OR.(J.EQ.12))DELE=1.0
GO TO 800
DELE=1.0
IF(KCOUNT.EQ.1)GO TO 98
IF(KCOUNT.EQ.2)GO TO 100
IF(MOD((K-1),NP))110,108,110
KK=1+(K-1)/20
DD(1,J,KK)=U(K)/U1
DD(2,J,KK)=ALPHA(K)
DD(3,J,KK)=Q(K)
DD(4,J,KK)=THETA(K)
GO TO 110
IF(MOD((K-1),NP))110,120,110
WRITE(2,*)U(K),ALPHA(K),Q(K),THETA(K)
FORMAT(4F10.4)
GO TO 110
WRITE(21,*)U(K),ALPHA(K),Q(K),THETA(K)
IF(MOD((K-1),NP))110,106,110
WRITE(3,*)U(K),ALPHA(K),Q(K),THETA(K)
SOLUTION OF EQUATIONS OF MOTIONS BY RUNGE-KUTTA METHOD.
CALL RUNKUT(UDOT,U(K),ALPHA(K),Q(K),THETA(K),U(K+1))
CALL RUNKUT(ALPDOT,ALPHA(K),U(K),Q(K),THETA(K),ALPHA(K+1))
CALL RUNKUT(QDOT,Q(K),U(K),ALPHA(K),THETA(K),Q(K+1))
THETA(K+1)=THETA(K)+Q(K)*TIMINC
T=T+TIMINC
CONTINUE
IF(KCOUNT.EQ.1)GO TO 1000
IF(KCOUNT.EQ.3)GO TO 125
IF(KCOUNT.EQ.2)GO TO 52
-----GENERATION OF PSEUDO NORMAL NUMBER -------------
IF(ITER.NE.1)GO TO 54
IF(NOISE.EQ.0)GO TO 54
N=49
CALL GGNOR(ISEED,N,R1)
CALL GGNOR(ISEED,N,R2)
CALL GGNOR(ISEED,N,R3)
CALL GGNOR(ISEED,N,R4)
```

```
READ(2,*)(U(I),ALPHA(I),Q(I),THETA(I),I=1,50)
DO 60 I=2,50
U(I)=U(I)+R1(I-1)*SIG(1)*PERN
ALPHA(I)=ALPHA(I)+R2(I-1)*SIG(2)*PERN
Q(I)=Q(I)+R3(I-1)*SIG(3)*PERN
THETA(I)=THETA(I)+R4(I-1)*SIG(4)*PERN
WRITE(6,61)(U(I),ALPHA(I),Q(I),THETA(I),I=1,50)
FORMAT(4E20.8)
KCOUNT=3
GO TO 33
-----------FORMATION OF MATRIX A -----------------------------
DO 130 I=1,4
DO 130 KK=1,50
II=50*(I-1)+KK
DO 130 JX=1,11
J=JX
IF(J.GE.3)J=J+1
A(II,JX)=PD(I,J,KK)
--------------------------------------------------------------
IF(INMAX.NE.1)GO TO 132
IF(ITER.NE.ITRMAX)GO TO 132
IF(NOISE.EQ.0)GO TO 132
------ESTIMATION OF CRAMER RAO BOUNDS --------------
ISGMA=VARIABLE USED FOR ESTIMATION OF CRAMER-RAO BOUNDS IN
TWO WAYS;IDP=TOTAL NO. OF DATA POINTS FOR EACH MEASURED VARIABLE
SMU,SMAL,SMQ,SMTH VARIABLES ARE USED FOR SUMMATION OF U,ALPHA
Q AND THETA MOTION VARIABLES.
ISGMA=1
IDP=50
IF(ISGMA.EQ.1)GO TO 442
PERN=1.0
REWIND(3)
REWIND(6)
READ(6,61)(UM(I),ALPHAM(I),QM(I),THETAM(I),I=1,IDP)
READ(3,*)(UE(I),ALPHAE(I),QE(I),THETAE(I),I=1,IDP)
SMU=0.0
DO 434 I=1,IDP
SMU=SMU+(UM(I)-UE(I))**2
SIG(1)=SQRT(SMU/FLOAT(IDP))
SMAL=0.0
DO 436 I=1,IDP
SMAL=SMAL+(ALPHAM(I)-ALPHAE(I))**2
SIG(2)=SQRT(SMAL/FLOAT(IDP))
SMQ=0.0
DO 438 I=1,IDP
SMQ=SMQ+(QM(I)-QE(I))**2
SIG(3)=SQRT(SMQ/FLOAT(IDP))
SMTH=0.0
DO 440 I=1,IDP
SMTH=SMTH+(THETAM(I)-THETAE(I))**2
SIG(4)=SQRT(SMTH/FLOAT(IDP))
S1,S2,S3,S4 ARE DIAGONAL ELEMENTS OF MATRIX D
  S1=(U1*U1)/((PERN*SIG(1))**2)
S2=1.0/((PERN*SIG(2))**2)
S3=1.0/((PERN*SIG(3))**2)
S4=1.0/((PERN*SIG(4))**2)
IMX=200;JMX=200;LMX=11
GO TO 510
IF(ISGMA.NE.1)GO TO 510
---CHECKING OF DIAGONAL ELEMENTS OF F MATRIX ----
DO 358 J=1,11
SUMU=0.0
DO 350 I=1,50
SUMU=SUMU+(A(I,J))**2
Y1=SUMU*S1
SUMAL=0.0
```

```
DO 352 I=51,100
SUMAL=SUMAL+(A(I,J))**2
Y2=SUMAL*S2
SUMQ=0.0
DO 354 I=101,150
SUMQ=SUMQ+(A(I,J))**2
Y3=SUMQ*S3
SUMTH=0.0
DO 356 I=151,200
SUMTH=SUMTH+(A(I,J))**2
Y4=SUMTH*S4
DEF(J)=Y1+Y2+Y3+Y4
CONTINUE
WRITE(5,401)(DEF(J),J=1,11)
FORMAT(5X,4E18.4)
WRITE(10,402)(DEF(J),J=1,11)
FORMAT(3X,'DEF'//(5X,4E18.4))
----GENERATION OF D MATRIX ------------------
DO 501 I=1,50
D(I,I)=S1
DO 502 I=51,100
D(I,I)=S2
DO 503 I=101,150
D(I,I)=S3
DO 504 I=151,200
D(I,I)=S4
IF(ISGMA.NE.1)GO TO 263
-------TRANSPOSE OF A MATRIX --------------
DO 262 I=1,IMX
DO 262 J=1,JMX
AT(J,I)=A(I,J)
CONTINUE
------MULTIPLICATION OF AT AND D MATRIX ------
DO 264 I=1,JMX
DO 264 J=1,IMX
E(I,J)=AT(I,J)*D(J,J)
CONTINUE
----MULTIPLICATION OF E AND A MATRIX ------------
DO 266 I=1,JMX
DO 266 K=1,JMX
F(I,K)=0.0
DO 268 I=1,JMX
DO 268 K=1,JMX
DO 268 J=1,IMX
SUM1=E(I,J)*A(J,K)
F(I,K)=F(I,K)+SUM1
CONTINUE
WRITE(5,401)(F(I,I),I=1,11)
WRITE(10,403)(F(I,I),I=1,11)
FORMAT(3X,'DIAGONAL ELEMENTS OF F MATRIX'//(5X,4E18.4))
---------INVERSION OF F MATRIX ---------------
N=11;IA1=11
IUNIT=11;IFAIL=0
CALL F01AAF(F,IA1,N,UNIT,IUNIT,WKSPCE,IFAIL)
WRITE(5,*)IFAIL
DO 410 I=1,11
CRB(I)=SQRT(UNIT(I,I))
WRITE(5,401)(CRB(I),I=1,11)
IF(ISGMA.NE.1)GO TO 444
WRITE(10,404)(CRB(I),I=1,11)
FORMAT(3X,'CRAMER RAO BOUNDS FROM MEASUREMENT NOISE'//
1    (5X,4E18.4))
GO TO 446
WRITE(10,405)(CRB(I),I=1,11)
FORMAT(3X,'CRAMER RAO BOUNDS FROM ESTIMATED RESPONSE'
1    /(5X,4E18.4))
```

```
ISGMA=ISGMA+1
IF(ISGMA.GT.2)GO TO 132
GO TO 448
-----FORMATION OF MATRIX B ------
REWIND(3)
IF(NOISE.EQ.0)GO TO 133
REWIND(6)
DO 140 I=1,50
READ(6,61)U6,ALPHA6,Q6,THETA6
READ(3,*)U3,ALPHA3,Q3,THETA3
B(I,1)=(U6-U3)/U1
B(I+50,1)=ALPHA6-ALPHA3
B(I+100,1)=Q6-Q3
B(I+150,1)=THETA6-THETA3
REWIND(6)
GO TO 137
REWIND(2)
DO 161 I=1,50
READ(2,*)U2,ALPHA2,Q2,THETA2
READ(3,*)U3,ALPHA3,Q3,THETA3
B(I,1)=(U2-U3)/U1
B(I+50,1)=ALPHA2-ALPHA3
B(I+100,1)=Q2-Q3
B(I+150,1)=THETA2-THETA3
CONTINUE
----- SOLUTION OF LINEAR SIMULTANEOUS EQUATIONS ---------
REWIND(2)
M=200;IA=200;IB=200;IDGT=0
NA=11
NB=1
CALL LLSQAR(A,B,M,NA,NB,IA,IB,IDGT,WKAREA,IER)
DO 145 I=1,11
X(I,1)=B(I,1)
-----UPDATING OF A/C AND STORE PARAMETERS ------
X0=X0+X(1,1)
XALPHA=XALPHA+X(2,1)
XDELE=CDS+X(3,1)
CDS=XDELE
Z0=Z0+X(4,1)
ZALPHA=ZALPHA+X(5,1)
ZQ=ZQ+X(6,1)
ZDELE=CLS+X(7,1)
CLS=ZDELE
M0=M0+X(8,1)
MALPHA=MALPHA+X(9,1)
MQ=MQ+X(10,1)
MDELE=CMS+X(11,1)
CMS=MDELE
WRITE(5,141)X0,XALPHA,XQ
WRITE(5,141)Z0,ZALPHA,ZQ
WRITE(5,141)M0,MALPHA,MQ
FORMAT(5X,3F19.4)
WRITE(10,142)ITER
FORMAT(10X,'ITERATION NO.',I3)
WRITE(10,144)X0,XALPHA,X0
FORMAT(5X,'X0=',F9.4,5X,'XALPHA=',F9.4,5X,'X0=',F9.4)
WRITE(10,143)Z0,ZALPHA,Z0
FORMAT(5X,'Z0=',F9.4,5X,'ZALPHA=',F9.4,5X,'Z0=',F9.4)
FORMAT(5X,'M0=',F9.4,5X,'MALPHA=',F9.4,5X,'M0=',F9.4)
WRITE(10,148)M0,MALPHA,M0
WRITE(5,*)CDS,CLS,CMS
WRITE(10,146)CDS,CLS,CMS
FORMAT(5X,'CDS=',F8.4,5X,'CLS=',F8.4,9X,'CMS=',F8.4)
ITER=ITER+1
WRITE(5,*)ITER
IF(ITER.GT.ITRMAX)GO TO 200
```

```
KCOUNT=2
REWIND(21)
REWIND(3)
GO TO 17
--- STORING OF A/C AND STORE PARAMETERS ----
DD(1,IN)=XU
DD(2,IN)=XALPHA
DD(3,IN)=ZU
DD(4,IN)=ZALPHA
DD(5,IN)=ZQ
DD(6,IN)=MU
DD(7,IN)=MALPHA
DD(8,IN)=MQ
DD(9,IN)=CDS
DD(10,IN)=CLS
DD(11,IN)=CMS
WRITE(5,*)IN
IN=IN+1
IF(IN.GT.INMAX)GO TO 210
REWIND(1)
REWIND(2)
GO TO 300
WRITE(10,25)((DD(JN,IN),IN=1,INMAX),JN=1,11)
FORMAT(3X,'A/C AND STORE PARAMETERS'//(5X,4E18.4))
IF(INMAX.EQ.1)GO TO 1000
CALCULATION OF MEAN AND SAMPLE STANDARD DEVIATION
DO 220 JN=1,11
SUM(JN)=0.0
DO 230 IN=1,INMAX
SUM(JN)=SUM(JN)+DD(JN,IN)
DDM(JN)=SUM(JN)/FLOAT(INMAX)
SMTN(JN)=0.0
DO 240 IN=1,INMAX
SMTN(JN)=SMTN(JN)+(DD(JN,IN)-DDM(JN))**2
CONTINUE
STDVN(JN)=SQRT(SMTN(JN)/FLOAT(INMAX-1))
CONTINUE
WRITE(10,26)(DDM(JN),JN=1,11)
FORMAT(5X,'MEAN VALUES OF AIRCRAFT AND STORE PARAMETERS'/
1 (7X,3E20.4))
WRITE(10,27)(STDVN(JN),JN=1,11)
FORMAT(5X,'SAMPLE STANDARD DEVIATION OF PARAMETER ESTIMATES'/
1  (7X,3E20.4))
STOP
CALL UPSDUR
CALL LSVALR
CALL UERTST
CALL VSORTP
CALL GGUB
CALL MERRI
END
-------------------------------------------------------------------
FUNCTION UDOT
FUNCTION UDOT(UX,ALPHAX,QX,THETAX)
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MIALPA,MALDOT,MQ,MDELE
COMMON/A3/ZU,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
UDOT=-G*THETAX*COS(THETA1)+(XU+XTU)*UX+XALPHA*ALPHAX+XQ*QX+XDELE
1     *DELE
RETURN
END
-------------------------------------------------------------------
FUNCTION ALPDOT
FUNCTION ALPDOT(ALPHAX,UX,QX,THETAX)
```

due to subroutine LLSQAR

```
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
ALPDOT=(-G*THETAX*SIN(THETA1)+ZU*UX+ZALPHA*ALPHAX+(ZQ+U1)*QX
1   +ZDELE*DELE)/(U1-ZALDOT)
RETURN
END
-----------------------------------------------------------------
FUNCTION QDOT
FUNCTION QDOT(QX,UX,ALPHAX,THETAX)
REAL MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
ALDOT=ALPDOT(ALPHAX,UX,QX,THETAX)
QDOT=(MU+MTU)*UX+(MALPHA+MTALPA)*ALPHAX+MALDOT*ALDOT+MQ*QX
1   +MDELE*DELE
RETURN
END
-----------------------------------------------------------------
SUBROUTINE FOR RUNGE KUTTA FOURTH ORDER METHOD
SUBROUTINE RUNKUT(FUN,VAR1,VAR2,VAR3,VAR4,VAR5)
COMMON/A5/H
S1=FUN(VAR1,VAR2,VAR3,VAR4)
S2=FUN(VAR1+0.5*H*S1,VAR2,VAR3,VAR4)
S3=FUN(VAR1+0.5*H*S2,VAR2,VAR3,VAR4)
S4=FUN(VAR1+S3*H,VAR2,VAR3,VAR4)
S=(S1+S2*2.0+S3*2.0+S4)/6.0
VAR5=VAR1+S*H
RETURN
END
*****************************************************************
```

```
********************
FILE:RSP.FOR
********************
PROGRAM FOR ESTIMATION OF DIMENSIONAL DRIVATIVES(PARAMETERS)OF
AN AIRCRAFT FROM SIMULATED FLIGHT TEST DATA USING GAUSS NEWTON
METHOD.
AIRCRAFT:FIAT WITHOUT ANY  STORE ;INPUT IS ELEVATOR
DEFLECTION (ARBITRARY).
NOTE:XQ IS NOT ESTIMATED;ZDELE HAS BEEN EESTIMATED THROUGH MDELE
XDELE IS NOT FREEZED
REAL MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE,MDEL1
IYY=PITCH MOMENT OF INERTIA OF A/C(KG-M**2);LT=DISTANCE BETWEEN
C.G. AND A.C.OF H.TAIL OF A/C(M);MA=MASS OF A/C(KG)
G=ACCELERATION DUE TO GRAVITY(M/SEC**2)
SUBSCRIPTED VARIABLES:TS=TIME(SEC);DELES=ELEVATOR DEFLECTION
(RAD);PD=PARTIAL DERIVATIVES;U,ALPHA,Q,THETA ARE MEASURED
VARIABLES;A=MATRIX CONTAINING PARTIAL DERIVATIVES ;B=MATRIX
CONTAINING ELEMENTS OF DIFFERENCE BETWEEN MEASURED/TRUE AND
ESTIMATED RESPONSE;X=MATRIX OF CORRECTION TERMS OF PARAMETERS
U21,ALFA21,Q21,TETA21 ARE MEASURED VARIABLES OF ESTIMATED
RESPONSE AT STEP SIZE OF 0.005 SEC;WKAREA=WORK AREA REQUIRED
FOR EXECUTION OF IMSL SUBROUTINE,LLSQAR;SIG=STANDARD DEVITION
OF MEASURED NOISE;R1,R2,R3,R4 ARE PSEUDO RANDOM NUMBERS
REAL IYY,LT,MA
DD=DIMENSIONAL DERIVATIVES;DDM=MEAN VALUES OF DIMENSIONAL
DERIVATIVES;STDVN=STANDARD DEVIATION;SUM=SUMMATION;MATRIX D
IS USED FOR ESTIMATION OF CRAMER-RAO BOUNDS;AT=TRANSPOSE OF
MATRIX A;E=MATRIX OBTAINED BY MULTIPLYING AT AND D;F=MATRIX
GIVES MULTIPLICATION OF MATRICES E AND A;UNIT=INVERSION OF
MATRIX F;WKSPCE=WORK SPACE USED IN NAG SUBROUTINE,F01AAF
DEF=DIOGONAL ELEMENTS OF F MATRIX;CRB=CRAMER-RAO BOUNDS
SUM AND SMTN VARIABLES ARE USED FOR SUMMATION
UDOT,ALPDOT,QDOT ARE DERIVATIVES OF U,ALPHA AND Q RESPECTIVELY
DIMENSION TS(21),DELES(21),PD(4,12,50),U(981),ALPHA(981)
DIMENSION Q(981),THETA(981),A(200,10),B(200,1),X(10,1)
DIMENSION U21(981),ALFA21(981),Q21(981),TETA21(981),WKAREA(200)
DIMENSION SIG(4),R1(49),R2(49),R3(49),R4(49)
DIMENSION DD(11,20),DDM(11),STDVN(11),SUM(11),SMTN(11)
DIMENSION D(200,200),AT(10,200),E(10,200),F(10,10)
DIMENSION UNIT(10,10),WKSPCE(13),DEF(10),CRB(10)
EXTERNAL UDOT,ALPDOT,QDOT
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
OPEN(UNIT=1,DEVICE='DSK',FILE='RSP.DAT',)
OPEN(UNIT=2,DEVICE='DSK',FILE='TRSP.OUT')
OPEN(UNIT=3,DEVICE='DSK',FILE='ERSP.OUT',)
OPEN(UNIT=6,DEVICE='DSK',FILE='MRSP.OUT')
OPEN(UNIT=[illegible],DEVICE='DSK',FILE='TPRSP.OUT')
OPEN(UNIT=9,DEVICE='DSK',FILE='DD.OUT')
OPEN(UNIT=20,DEVICE='DSK',FILE='P.DAT')
TOTTIM=TOTAL TIME; CONTIM=CONTROL TIME; TIMINC=TIME INCREAMENT
U1=STEADY STATE SPEED; INPTS=INPUT POINTS; NTVALUE INDICATES
TYPE OF INPUT; NT=1 MEANS INPUT IS SIN FUNCTION; NT=2MEANS
STEP INPUT; NT=3 MEANS INPUT IS HALF PULSE OR FULL PULSE
OR ARBITRARY;KCOUNT=1 GIVES TRUE(NO-NOISE) RESPONSE
KCOUNT=2 IS FOR ESTIMATED(MODEL) AND NOISY (MEASURED)RESPONSE
KCOUNT=3 IS USED FOR ESTIMATION OF PARTIAL DDERIVATIVES AND
PARAMETERS OF A/C.
IN=NOISE SAMPLE;ITER=ITERATION NUMBER;ITRMAX=MAXIMUM NO. OF
ITERATIONS ALLOWED;ISEED=NUMBER ASSIGNED FOR GENERATION OF
RANDOM NUMBERS;PERC=PERCENTAGE NOISE.
 IN=1
 ITER=1
```

```
READ(1,*)XU,XTU,XALPHA,XQ,XDELE
READ(1,*)ZU,ZALPHA,ZALDOT,ZQ,ZDELE
READ(1,*)MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
READ(1,*)TOTTIM,CONTIM,TIMINC,U1,NT,KCOUNT,ITRMAX
READ(1,*)IYY,LT,MA
IF(KCOUNT.EQ.1)GO TO 4
READ(1,*)PERN,SIG(1),SIG(2),SIG(3),SIG(4)
READ(1,*)ISEED,INMAX,NOISE
IF(IN.NE.1)GO TO 8
GO TO 4
ISEED=ISEED+IN*57255
XDEL1=XDELE
ZDEL1=ZDELE
MDEL1=MDELE
INPTS=1.0+CONTIM/0.1
IF(NT.EQ.3)GO TO 45
GO TO 55
--------------------------------------------------------------
IF(IN.NE.1)GO TO 6
READ(20,*)(TS(J),DELES(J),J=1,INPTS)
WRITE(9,16)
FORMAT(6X,'TS',8X,'DELES',6X,'TS',8X,'DELES')
WRITE(9,17)(TS(J),DELES(J),J=1,INPTS)
FORMAT(5X,F4.2,5X,F7.4,5X,F4.2,5X,F7.4)
--------------------------------------------------------------
-----WRITE INPUT VALUES ----
IF(IN.NE.1)GO TO 6
WRITE(9,11)
FORMAT(//1X,'INPUT:'/4X,'STARTING DIMENSIONAL DERIVATIVES:')
WRITE(9,12)XU,XTU,XALPHA,XQ,XDELE
FORMAT(/6X,'XU=',E14.6/6X,'XTU=',E14.6/6X,'XALPHA=',E14.6/
1    6X,'XQ=',E14.6/6X,'XDELE=',E14.6)
WRITE(9,13)ZU,ZALPHA,ZALDOT,ZQ,ZDELE
FORMAT(/6X,'ZU=',E14.6/6X,'ZALPHA=',E14.6/6X,'ZALDOT=',E14.6/
1    6X,'ZQ=',E14.6/6X,'ZDELE=',E14.6)
WRITE(9,14)MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
FORMAT(/6X,'MU=',E14.6/6X,'MTU=',E14.6/6X,'MALPHA=',E14.6/
1    6X,'MTALPA=',E14.6/6X,'MALDOT=',E14.6/6X,'MQ=',E14.6/
1    6X,'MDELE=',E14.6)
WRITE(9,15)TOTTIM,CONTIM,TIMINC,U1,NT,KCOUNT,ITRMAX
FORMAT(/6X,'TOTAL TIME=',E14.6/6X,'CONTROL TIME=',E14.6/
1    6X,'TIME INCREAMENT=',E14.6/6X,'U1=',E14.6,5X,'NT=',I2
1 /  6X,'KCOUNT=',I2,5X,'ITRMAX=',I2)
WRITE(9,19)IYY,LT,MA
FORMAT(5X,'IYY=',F9.2,5X,'LT=',F8.4,5X,'MA=',F8.2)
IF(KCOUNT.EQ.1)GO TO 6
WRITE(9,18)PERN,SIG(1),SIG(2),SIG(3),SIG(4),ISEED,INMAX,NOISE
FORMAT(5X,'PERN=',F5.2,5X,'SIG(1)=',F8.4,5X,'SIG(2)='
1    ,F8.5/5X,'SIG(3)=',F8.5,5X,'SIG(5)=',F8.5/5X,'ISEED=',I9
1    ,5X,'INMAX=',I3,5X,'NOISE=',I3)
--------------------------------------------------------------
CONTINUE
-----INITIAL FLIGHT CONDITIONS ----------
DATA THETA(1)/0.0/,U(1)/0.0/,Q(1)/0.0/,THETA1/0.0/
DATA ALFHA(1)/0.0/,G/9.81/,PI/3.1416/
--------------------------------------------------------------
KTOT=NUMBER OF POINTS FOR WHICH RESPONSE IS CALCULATED.
KCON=TOTAL NO. OF CONTROL TIME POINTS AT WHICH ELEVATOR
DEFLECTION IS KNOWN;NP=VARIABLE USED TO SCRAMBLE BACK THE
RESPONSES AT STEP SIZE OF 0.1 SECOND;T=TIME(SEC)
KTOT=981
KCON=CONTIM/TIMINC+1.0
NP=0.1/TIMINC
GO TO 34
REWIND(21)
READ(21,*)(U21(K),ALFA21(K),Q21(K),TETA21(K),K=1,KTOT)
```

```
JCOUNT=12
IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))JCOUNT=1
DO 50 J=1,JCOUNT
T=0.0
L=1
IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))GO TO 35
-----ESTIMATION OF PARTIAL DERIVATIVES ----------
IF((J.EQ.1).OR.(J.EQ.2).OR.(J.EQ.3).OR.(J.EQ.4))GO TO 610
IF((J.EQ.5).OR.(J.EQ.6).OR.(J.EQ.7).OR.(J.EQ.8))GO TO 620
XDELE=0.0
ZDELE=0.0
MDELE=1.0
GO TO35
XDELE=1.0
ZDELE=0.0
MDELE=0.0
GO TO 35
XDELE=0.0
ZDELE=1.0
MDELE=0.0
DO 50 K=1,KTOT
IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))GO TO 700
IF((J.EQ.1).OR.(J.EQ.5).OR.(J.EQ.9))DELE=U21(K)
IF((J.EQ.2).OR.(J.EQ.6).OR.(J.EQ.10))DELE=ALFA21(K)
IF((J.EQ.3).OR.(J.EQ.7).OR.(J.EQ.11))DELE=Q21(K)
IF((J.EQ.4).OR.(J.EQ.8).OR.(J.EQ.12))GO TO 700
GO TO 800
IF(K.GT.KCON)GO TO 90
GO TO (75,85,95),NT
-----------FOR SIN FUNCTION INPUT ----------------------------
DELE=0.1*SIN(PI*T)
GO TO 800
-----------FOR STEP INPUT -------------------------------------
IF(T.EQ.0.0) GO TO 65
DELE=0.1
GO TO 800
DELE=0.0
GO TO 800
-------------------------------------------------------------------
---CALCULATION OF ELEVATOR DEFLECTION AT INTERMEDIATE -----
-----TIME POINTS THROUGH INTERPOLATION --------
IF(T.EQ.TS(L))GO TO 30
IF((T.GT.TS(L)).AND.(T.LT.TS(L+1)))GO TO 40
IF(T.EQ.TS(L+1))GO TO 80
L=L+1
DELE=DELES(L)+(DELES(L+1)-DELES(L))*(T-TS(L))/(TS(L+1)-TS(L))
GO TO 800
DELE=DELES(L)
GO TO 800
DELE=DELES(L+1)
GO TO 800
IF(NT.EQ.2)GO TO 105
DELE=0.0
IF(KCOUNT.EQ.1)GO TO 98
IF(KCOUNT.EQ.2)GO TO 100
IF(MOD((K-1),NP))110,108,110
KK=1+(K-1)/2
PD(1,J,KK)=U(K)/U1
PD(2,J,KK)=ALPHA(K)
PD(3,J,KK)=Q(K)
PD(4,J,KK)=THETA(K)
GO TO 110
IF(MOD((K-1),NP))110,120,110
WRITE(2,*)U(K),ALPHA(K),Q(K),THETA(K)
FORMAT(4F12.4)
GO TO 110
```

```
WRITE(21,*)U(K),ALPHA(K),Q(K),THETA(K)
IF(MOD((K-1),NP))110,106,110
WRITE(3,*)U(K),ALPHA(K),Q(K),THETA(K)
SOLUTION OF GOVERNING EQUATIONS OF MOTION BY USING RUNGE
KUTTA FOURTH ORDER METHOD
CALL RUNKUT(UDOT,U(K),ALPHA(K),Q(K),THETA(K),U(K+1))
CALL RUNKUT(ALPDOT,ALPHA(K),U(K),Q(K),THETA(K),ALPHA(K+1))
CALL RUNKUT(QDOT,Q(K),U(K),ALPHA(K),THETA(K),Q(K+1))
THETA(K+1)=THETA(K)+Q(K)*TIMINC
T=T+TIMINC
CONTINUE
IF(KCOUNT.EQ.1)GO TO 1000
IF(KCOUNT.EQ.3)GO TO 125
IF(KCOUNT.EQ.2)GO TO 52
----GENERATION OF PSEUDO NORMAL RANDOM NUMBER ------------------
N=RANDOM  NUMBERS  TO BE GENERATED FOR EACH MEASURED VARIABLE
R IS OUTPUT VECTOR CONTAINING THE NORMAL PSEUDO RANDOM NUMBERS
ISEED IS INPUT.AN INTEGER VALUE IN THE EXCLUSIVE RANGE
(1,2147483647).ISEED IS REPLACED BY A NEW ISEED TO BE
USED IN SUBSEQUENT CALLS
IF(ITER.NE.1)GO TO 54
N=49
CALL GGNOR(ISEED,N,R1)
CALL GGNOR(ISEED,N,R2)
CALL GGNOR(ISEED,N,R3)
CALL GGNOR(ISEED,N,R4)
READ(2,*)(U(I),ALPHA(I),Q(I),THETA(I),I=1,50)
DO 60 I=2,50
U(I)=U(I)+R1(I-1)*SIG(1)*PERN
ALPHA(I)=ALPHA(I)+R2(I-1)*SIG(2)*PERN
Q(I)=Q(I)+R3(I-1)*SIG(3)*PERN
THETA(I)=THETA(I)+R4(I-1)*SIG(4)*PERN
WRITE(6,61)(U(I),ALPHA(I),Q(I),THETA(I),I=1,50)
FORMAT(4E20.8)
KCOUNT=3
GO TO 33
------------FORMATION OF MATRIX A ---------------------------
DO 130 I=1,4
DO 130 KK=1,50
II=50*(I-1)+KK
DO 130 JX=1,15
J=JX
IF(J.GE.3)J=J+1
IF(J.GE.8)J=J+1
A(II,JX)=PD(I,J,KK)
-------------------------------------------------------------
IF(NOISE.EQ.5)GO TO 132
IF(IEMAX.NE.1)GO TO 132
IF(ITER.NE.ITRMAX)GO TO 132
-----ESTIMATION OF CRAMER RAO BOUNDS -------------
S1=(U1*U1)/((PERN*SIG(1))**2)
S2=1.0/((PERN*SIG(2))**2)
S3=1.0/((PERN*SIG(3))**2)
S4=1.0/((PERN*SIG(4))**2)
IMX=200;JMX=200;LMX=10
GO TO 510
---CHECKING OF DIAGONAL ELEMENTS OF F MATRIX ---
DO 358 J=1,10
SUMU=0.0
DO 350 I=1,50
SUMU=SUMU+(A(I,J))**2
Y1=SUMU*S1
SUMAL=0.0
DO 352 I=51,100
SUMAL=SUMAL+(A(I,J))**2
Y2=SUMAL*S2
```

```
SUMQ=0.0
DO 354 I=101,150
SUMQ=SUMQ+(A(I,J))**2
Y3=SUMQ*S3
SUMTH=0.0
DO 356 I=151,200
SUMTH=SUMTH+(A(I,J))**2
Y4=SUMTH*S4
DEF(J)=Y1+Y2+Y3+Y4
CONTINUE
WRITE(5,401)(DEF(J),J=1,10)
FORMAT(5X,4E18.4)
WRITE(9,402)(DEF(J),J=1,10)
FORMAT(3X,'DEF'//(5X,4E18.4))
----GENERATION OF D MATRIX ------------------
DO 501 I=1,50
D(I,I)=S1
DO 502 I=51,100
D(I,I)=S2
DO 503 I=101,150
D(I,I)=S3
DO 504 I=151,200
D(I,I)=S4
-------TRANSPOSE OF A MATRIX ----------------
DO 262 I=1,IMX
DO 262 J=1,LMX
AT(J,I)=A(I,J)
CONTINUE
------MULTIPLICATION OF AT AND D MATRIX ------
DO 264 I=1,LMX
DO 264 J=1,JMX
E(I,J)=AT(I,J)*D(J,J)
CONTINUE
---- MULTIPLICATION OF E AND A MATRIX ------------
DO 266 I=1,LMX
DO 266 K=1,LMX
F(I,K)=0.0
DO 268 I=1,LMX
DO 268 K=1,LMX
DO 268 J=1,JMX
SUM1=E(I,J)*A(J,K)
F(I,K)=F(I,K)+SUM1
CONTINUE
WRITE(5,401)(F(I,I),I=1,10)
WRITE(9,403)(F(I,I),I=1,10)
FORMAT(3X,'DIAGONAL ELEMENTS OF F MATRIX'//(5X,4E18.4))
---------INVERSION OF F MATRIX ---------------
N=ORDER OF MATRIX F;IA1=FIRST DIMENSION OF ARRAY F
IA1 SHOULD BE .GE.N;UNIT=REAL ARRAY OF DIMENSION(IUNIT,P)
WHERE P SHOULD BE .GE. N;ON EXIT UNIT CONTAINS THE INVERSE
OF F;IUNIT=INTEGER AND SPECIFIES THE FIRST DIMENSION OF
ARRAY UNIT.IUNIT SHOULD BE .GE.N;WKSPCE IS REAL ARRAY OF
DIMENSION AT LEAST(N).USED AS WORKING SPACE;IFAIL
IS ASSIGNED A VALUE 0.UNLESS THE ROUTINE DETECTS AN ERROR
IFAIL CONTAINS 0 ON EXIT
N=10;IA1=10
IUNIT=10;IFAIL=0
CALL F01AAF(F,IA1,N,UNIT,IUNIT,WKSPCE,IFAIL)
WRITE(5,*)IFAIL
DO 410 I=1,10
CRB(I)=SQRT(UNIT(I,I))
WRITE(5,401)(CRB(I),I=1,10)
WRITE(9,404)(CRB(I),I=1,10)
FORMAT(3X,'CRAMER RAO BOUNDS'//(5X,4E18.4))
-------------------------------------------------------------
-----FORMATION OF MATRIX G-----------------------
```

```
REWIND(3)
REWIND(6)
DO 140 I=1,50
READ(6,61)UT,ALPHAT,QT,THETAT
READ(3,*)UE,ALPHAE,QE,THETAE
B(I,1)=(UT-UE)/U1
B(I+50,1)=ALPHAT-ALPHAE
B(I+100,1)=QT-QE
B(I+150,1)=THETAT-THETAE
REWIND(6)
----SOLUTION OF LINEAR SIMULTANEOUS EQUATIONS----------
M=NO. OF ROWS IN MATRICES A AND B;IA=ROW DIMENSION OF
MATRIX A IN THE CALLING PROGRAM;IB=ROW DIMENSION OF
MATRIX B IN THE CALLING PROGRAM;IDGT SPECIFIES SIGNIFICANT
DIGITS FOR ELEMENTS OF MATRIX A;NA=NO. OF COLUMNS IN
MATRIX A;NB=NO. OF COLUMNS IN MATRIX B;A IS THE COEFFICIENT
MATRIX OF EQN. AX=B WHEN A IS M*NA WITH M.GE.NA
B IS THE MATRIX OF THE R.H.S.OF THE EQN. AX=B WHERE B
IS M*NB
M=200;IA=200;IB=200;IDGT=6
NA=10
NB=1
CALL LLSQAR(A,B,M,NA,NB,IA,IB,IDGT,WKAREA,IER)
DO 145 I=1,10
X(I,1)=B(I,1)
---------UPDATING OF A/C PARAMETERS-------------
XU=XU+X(1,1)
XALPHA=XALPHA+X(2,1)
XDELE=XDEL1+X(3,1)
XDEL1=XDELE
ZU=ZU+X(4,1)
ZALPHA=ZALPHA+X(5,1)
ZQ=ZQ+X(6,1)
MU=MU+X(7,1)
MALPHA=MALPHA+X(8,1)
MQ=MQ+X(9,1)
MDELE=MDEL1+X(10,1)
MDEL1=MDELE
ZDELE=(MDELE*IYY)/(LT*MA)
WRITE(5,141)XU,XALPHA,XQ,XDELE
WRITE(5,141)ZU,ZALPHA,ZQ,ZDELE
WRITE(5,141)MU,MALPHA,MQ,MDELE
FORMAT(3X,4F19.4)
WRITE(5,*)ITER
WRITE(9,142)ITER
FORMAT(10X,'ITERATION NO.',I3)
WRITE(9,141)XU,XALPHA,XQ,XDELE
WRITE(9,141)ZU,ZALPHA,ZQ,ZDELE
WRITE(9,141)MU,MALPHA,MQ,MDELE
ITER=ITER+1
IF(ITER.GT.ITMAX)GO TO 200
KCOUNT=2
REWIND(10)
REWIND(3)
GO TO 34
-------STORING OF A/C PARAMETERS --------
DD(1,IN)=XU
DD(2,IN)=XALPHA
DD(3,IN)=XDELE
DD(4,IN)=ZU
DD(5,IN)=ZALPHA
DD(6,IN)=ZQ
DD(7,IN)=ZDELE
DD(8,IN)=MU
DD(9,IN)=MALPHA
DD(10,IN)=MQ
```

```
DD(11,IN)=MDELE
WRITE(5,*)IN
IN=IN+1
IF(IN.GT.INMAX)GO TO 210
REWIND(1)
REWIND(2)
GO TO 300
WRITE(9,25)((DD(JN,IN),IN=1,INMAX),JN=1,11)
FORMAT(3X,'AIRCRAFT PARAMETERS'//
1    (5X,4E18.4))
IF(INMAX.EQ.1)GO TO 1000
-----CALCULATION OF  MEAN AND SAMPLE  STANDARD DEVIATION -------
DO 220 JN=1,11
SUM(JN)=0.0
DO 230 IN=1,INMAX
SUM(JN)=SUM(JN)+DD(JN,IN)
DDM(JN)=SUM(JN)/FLOAT(INMAX)
SMTN(JN)=0.0
DO 240 IN=1,INMAX
SMTN(JN)=SMTN(JN)+(DD(JN,IN)-DDM(JN))**2
CONTINUE
STDVN(JN)=SQRT(SMTN(JN)/FLOAT(INMAX-1))
CONTINUE
WRITE(9,26)(DDM(JN),JN=1,11)
FORMAT(5X,'MEAN VALUES OF AIRCRAFT PARAMETERS'//(5X,3E20.4))
1   COEFFECIENTS'/(7X,3E20.4))
WRITE(9,27)(STDVN(JN),JN=1,11)
FORMAT(5X,'SAMPLE STANDARD DEVIATION OF PARAMETER ESTIMATES'/
1   (7X,3E20.4))
STOP
CALL DPSDUR
CALL DSVALE
CALL DERTST
CALL FSORTM
CALL GGUB
CALL MERFI
END
-------------------------------------------------------------------
FUNCTION UDOT
FUNCTION UDOT(UX,ALPHAX,QX,THETAX)
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
UDOT=-G*THETAX*COS(THETA1)+(XU+XTU)*UX+XALPHA*ALPHAX+XQ*QX+XDELE
1     *DELE
RETURN
END
-------------------------------------------------------------------
FUNCTION ALPDOT
FUNCTION ALPDOT(ALPHAX,UX,QX,THETAX)
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
ALPDOT=(-G*THETAX*SIN(THETA1)+ZU*UX+ZALPHA*ALPHAX+(ZQ+U1)*QX
1   +ZDELE*DELE)/(U1-ZALDOT)
RETURN
END
-------------------------------------------------------------------
FUNCTION QDOT
FUNCTION QDOT(QX,UX,ALPHAX,THETAX)
REAL MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
```

```
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/UI,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
ALDOT=ALPDOT(ALPHAX,UX,QX,THETAX)
QDOT=(MU+MTU)*UX+(MALPHA+MTALPA)*ALPHAX+MALDOT*ALDOT+MQ*QX
1    +MDELE*DELE
RETURN
END
-----------------------------------------------------------------
SUBROUTINE FOR RUNGE KUTTA FOURTH ORDER METHOD
SUBROUTINE RUNKUT(FUN,VAR1,VAR2,VAR3,VAR4,VAR5)
COMMON/A5/H
S1=FUN(VAR1,VAR2,VAR3,VAR4)
S2=FUN(VAR1+0.5*H*S1,VAR2,VAR3,VAR4)
S3=FUN(VAR1+0.5*H*S2,VAR2,VAR3,VAR4)
S4=FUN(VAR1+S3*H,VAR2,VAR3,VAR4)
S=(S1+S2*2.0+S3*2.0+S4)/6.0
VAR5=VAR1+S*H
RETURN
END
*****************************************************************
```

```
*****************
FILE:STR3.FOR
*****************
PROGRAM FOR ESTIMATION OF LONGITUDINAL AERODYNAMIC COEFFICIENTS
OF  STORE FROM FLIGHT TEST DATA
ALL PARAMETERS OF AIRCRAFT ARE FREEZED AT THEIR ESTIMATED VALUES
ONLY STORE PARAMETERS:CDS,CLS,CMS ARE ESTIMATED.THIS IS METHOD 2
REAL IYYA,MA,MS,NS
REAL MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE,MDEL1
DIMENSION PD(4,12,50),U(981),ALPHA(981)
DIMENSION Q(981),THETA(981),A(200,3),B(200,1),X(3,1)
DIMENSION U21(981),ALFA21(981),Q21(981),TETA21(981),WKAREA(200)
DIMENSION SIG(4),R1(49),R2(49),R3(49),R4(49)
DIMENSION DD(3,20),DDM(3),STDVN(3),SUM(3),SMTN(3)
DIMENSION D(200,200),AT(3,200),E(3,200),F(3,3)
DIMENSION UNIT(3,3),WKSPCE(13),DEF(3),CRB(3)
DIMENSION UM(50),ALPHAM(50),QM(50),THETAM(50)
DIMENSION UE(50),ALPHAE(50),QE(50),THETAE(50)
EXTERNAL UDOT,ALPDOT,QDOT
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
OPEN(UNIT=1,DEVICE='DSK',FILE='RSP.DAT',)
OPEN(UNIT=2,DEVICE='DSK',FILE='TRSP.OUT')
OPEN(UNIT=3,DEVICE='DSK',FILE='ERSP.OUT',)
OPEN(UNIT=6,DEVICE='DSK',FILE='MRSP.OUT')
OPEN(UNIT=21,DEVICE='DSK',FILE='TPRSP.OUT')
OPEN(UNIT=10,DEVICE='DSK',FILE='RSP.OUT')
IN=1
ITER=1
READ(1,*)TOTTIM,TIMINC,U1,KCOUNT,NOISE,INMAX,ITRMAX
READ(1,*)THETA1,RHO,G,SCF
READ(1,*)MA,VHS,S,CD1,IYYA,XCG
IF((KCOUNT.EQ.1).OR.(NOISE.EQ.0))GO TO 310
READ(1,*)PERN,SIG(1),SIG(2),SIG(3),SIG(4),ISEED
READ(1,*)XU,XTU,XALPHA,XQ,XDELE
READ(1,*)ZU,ZALPHA,ZALDOT,ZQ,ZDELE
READ(1,*)MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
READ(1,*)MS,CDS,CLS,CMS,SS,CS,DS,NS,XS
IF(IN.NE.1)GO TO 8
GO TO 10
ISEED=ISEED+IN*57255
GO TO 6
-----------------------------------------------------------------
------WRITE  INPUT VALUES ------
WRITE(10,11)
FORMAT(//1X,'INPUT:'/4X,'STARTING PARAMETER VALUES:')
WRITE(10,12)XU,XTU,XALPHA,XQ,XDELE
FORMAT(/6X,'XU=',E14.6/6X,'XTU=',E14.6/6X,'XALPHA=',E14.6/
1    6X,'XQ=',E14.6/6X,'XDELE=',E14.6)
WRITE(10,13)ZU,ZALPHA,ZALDOT,ZQ,ZDELE
FORMAT(/6X,'ZU=',E14.6/6X,'ZALPHA=',E14.6/6X,'ZALDOT=',E14.6/
1  6X,'ZQ=',E14.6/6X,'ZDELE=',E14.6)
WRITE(10,14)MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
FORMAT(/6X,'MU=',E14.6/6X,'MTU=',E14.6/6X,'MALPHA=',E14.6/
1    6X,'MTALPA=',E14.6/6X,'MALDOT=',E14.6/6X,'MQ=',E14.6/
1    6X,'MDELE=',E14.6)
WRITE(10,15)TOTTIM,TIMINC,U1,KCOUNT,NOISE,INMAX,ITRMAX
FORMAT(/6X,'TOTAL TIME=',E14.6/
1    6X,'TIME INCREAMENT=',E14.6/6X,'U1=',E14.6,5X,
1   'KCOUNT='I2,5X,'NOISE=',I3,5X,'INMAX=',I3,5X,'ITRMAX=',I3)
WRITE(10,20)MA,VHS,S,CD1,IYYA,XCG
FORMAT(3X,'MA=',F10.2,3X,'VHS=',F7.4,3X,'S=',F7.2,3X,'CD1=',
1  F7.4 ,3X,'IYYA=',F10.2,3X,'XCG=',F7.4)
```

```
      WRITE(10,21)MS,CDS,CLS,CMS,SS,CS,DS,NS,XS
      FORMAT(3X,'MS=',F6.1,3X,'CDS=',F7.4,3X,'CLS=',F7.4,3X,'CMS=',
     1 F8.4,3X,'SS=' ,F5.2/3X,'CS=',F5.2,3X,'DS=',F5.2,3X,'NS=',F4.1,
     1    3X,'XS=',F7.4)
      WRITE(10,22)THETA1,RHO,G,SCF
      FORMAT(3X,'THETA1=',F5.2,3X,'RHO=',F6.3,3X,'G=',F5.2,
     1   3X,'SCF=',F6.3)
      IF((KCOUNT.EQ.1).OR.(NOISE.EQ.0))GO TO 6
      WRITE(10,16)PERN,SIG(1),SIG(2),SIG(3),SIG(4),ISEED
      FORMAT(5X,'PERN=',F5.2,5X,'SIG(1)=',F10.6,5X,'SIG(2)=',F10.6/
     1    5X,'SIG(3)=',F10.6,5X,'SIG(4)=',F10.6,5X,'ISEED=',I12)
-----------------------------------------------------------------
-----INITIAL FLIGHT CONDITIONS -----
      CONTINUE
      DATA THETA(1)/0.0/,U(1)/0.0/,Q(1)/0.0/
      DATA ALPHA(1)/0.0/,PI/3.1416/
-----------------------------------------------------------------
      KTOT=981
      NP=0.1/TIMINC
      AF=3.1416*DS*DS/4.0
      QB=0.5*RHO*U1*U1
      QS=SCF*QB
      DT=-VHS*(MS/MA)
-----ESTIMATION OF STEP INPUTS -----
      AX=(CDS*QS*AF*NS+MS*G*SIN(THETA1))/MA
      AZ=(CLS*QS*SS*NS-MS*G*COS(THETA1))/MA
      AM=((-CMS*CS-(XCG-XS)*CLS)*QS*SS*NS+CDS*QS*AF*NS*(VHS-DT))/IYYA
      WRITE(10,18)AX,AZ,AM
      FORMAT(5X,'AX=',F8.5,5X,'AZ=',F8.5,5X,'AM=',F8.5)
      XDELE=AX
      ZDELE=AZ
      MDELE=AM
      GO TO 34
      REWIND(21)
      READ(21,*)(U21(K),ALFA21(K),Q21(K),TETA21(K),K=1,KTOT)
      JCOUNT=12
      IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))JCOUNT=1
      E1=QS*AF*NS
      E2=QS*SS*NS
      DO 50 J=1,JCOUNT
      T=0.0
      IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))GO TO 35
      IF((J.EQ.1).OR.(J.EQ.2).OR.(J.EQ.3))GO TO 610
      IF(J.EQ.4)GO TO 611
      IF((J.EQ.5).OR.(J.EQ.6).OR.(J.EQ.7))GO TO 620
      IF(J.EQ.8)GO TO 621
      IF(J.EQ.12)GO TO 622
      XDELE=0.0
      ZDELE=0.0
      MDELE=1.0
      GO TO35
      XDELE=1.0
      ZDELE=0.0
      MDELE=0.0
      GO TO 35
      XDELE=E1/MA
      ZDELE=0.0
      MDELE=E1*(VHS-DT)/IYYA
      GO TO 35
      XDELE=0.0
      ZDELE=1.0
      MDELE=0.0
      GO TO 35
      XDELE=0.0
      ZDELE=E2/MA
      MDELE=-(XCG-XS)*E2/IYYA
```

```
GO TO 35
XDELE=0.0
ZDELE=0.0
MDELE=-E2*CS/IYYA
DO 50 K=1,KTOT
IF((KCOUNT.EQ.1).OR.(KCOUNT.EQ.2))GO TO 700
IF((J.EQ.1).OR.(J.EQ.5).OR.(J.EQ.9))DELE=U21(K)
IF((J.EQ.2).OR.(J.EQ.6).OR.(J.EQ.10))DELE=ALFA21(K)
IF((J.EQ.3).OR.(J.EQ.7).OR.(J.EQ.11))DELE=Q21(K)
IF((J.EQ.4).OR.(J.EQ.8).OR.(J.EQ.12))DELE=1.0
GO TO 800
DELE=1.0
IF(KCOUNT.EQ.1)GO TO 98
IF(KCOUNT.EQ.2)GO TO 100
IF(MOD((K-1),NP))110,108,110
KK=1+(K-1)/20
PD(1,J,KK)=U(K)/U1
PD(2,J,KK)=ALPHA(K)
PD(3,J,KK)=Q(K)
PD(4,J,KK)=THETA(K)
GO TO 110
IF(MOD((K-1),NP))110,120,110
WRITE(2,*)U(K),ALPHA(K),Q(K),THETA(K)
FORMAT(4F12.4)
GO TO 110
WRITE(21,*)U(K),ALPHA(K),Q(K),THETA(K)
IF(MOD((K-1),NP))110,106,110
WRITE(3,*)U(K),ALPHA(K),Q(K),THETA(K)
SOLUTION OF EQUATIONS OF MOTION BY RUNGE-KUTTA METHOD.
CALL RUNKUT(UDOT,U(K),ALPHA(K),Q(K),THETA(K),U(K+1))
CALL RUNKUT(ALPDOT,ALPHA(K),U(K),Q(K),THETA(K),ALPHA(K+1))
CALL RUNKUT(QDOT,Q(K),U(K),ALPHA(K),THETA(K),Q(K+1))
THETA(K+1)=THETA(K)+Q(K)*TIMINC
T=T+TIMINC
CONTINUE
IF(KCOUNT.EQ.1)GO TO 1000
IF(KCOUNT.EQ.3)GO TO 125
IF(KCOUNT.EQ.2)GO TO 52
----GENERATION OF PSEUDO NORMAL NUMBER ------------
IF(ITER.NE.1)GO TO 54
IF(NOISE.EQ.0)GO TO 54
N=49
CALL GGNOR(ISEED,N,R1)
CALL GGNOR(ISEED,N,R2)
CALL GGNOR(ISEED,N,R3)
CALL GGNOR(ISEED,N,R4)
READ(2,*)(U(I),ALPHA(I),Q(I),THETA(I),I=1,50)
DO 60 I=2,50
U(I)=U(I)+R1(I-1)*SIG(1)*PERN
ALPHA(I)=ALPHA(I)+R2(I-1)*SIG(2)*PERN
Q(I)=Q(I)+R3(I-1)*SIG(3)*PERN
THETA(I)=THETA(I)+R4(I-1)*SIG(4)*PERN
WRITE(6,61)(U(I),ALPHA(I),Q(I),THETA(I),I=1,50)
FORMAT(4E20.8)
KCOUNT=3
GO TO 33
----------FORMATION OF MATRIX A --------------------------
DO 130 I=1,4
DO 130 KK=1,50
II=50*(I-1)+KK
DO 130 JX=1,3
J=JX
IF(J.GE.1)J=J+3
IF(J.GE.5)J=J+3
IF(J.GE.9)J=J+3
A(II,JX)=PD(I,J,KK)
```

```
C-------------------------------------------------------------------
      IF(INMAX.NE.1)GO TO 132
      IF(ITER.NE.ITRMAX)GO TO 132
      IF(NOISE.EQ.0)GO TO 132
C-----ESTIMATION OF CRAMER RAO BOUNDS -------------
      ISGMA=1
      IDP=50
      IF(ISGMA.EQ.1)GO TO 442
      PERN=1.0
      REWIND(3)
      REWIND(6)
      READ(6,61)(UM(I),ALPHAM(I),QM(I),THETAM(I),I=1,IDP)
      READ(3,*)(UE(I),ALPHAE(I),QE(I),THETAE(I),I=1,IDP)
      SMU=0.0
      DO 434 I=1,IDP
      SMU=SMU+(UM(I)-UE(I))**2
      SIG(1)=SQRT(SMU/FLOAT(IDP))
      SMAL=0.0
      DO 436 I=1,IDP
      SMAL=SMAL+(ALPHAM(I)-ALPHAE(I))**2
      SIG(2)=SQRT(SMAL/FLOAT(IDP))
      SMQ=0.0
      DO 438 I=1,IDP
      SMQ=SMQ+(QM(I)-QE(I))**2
      SIG(3)=SQRT(SMQ/FLOAT(IDP))
      SMTH=0.0
      DO 440 I=1,IDP
      SMTH=SMTH+(THETAM(I)-THETAE(I))**2
      SIG(4)=SQRT(SMTH/FLOAT(IDP))
          S1=(U1*U1)/((PERN*SIG(1))**2)
      S2=1.0/((PERN*SIG(2))**2)
      S3=1.0/((PERN*SIG(3))**2)
      S4=1.0/((PERN*SIG(4))**2)
      IMX=200;JMX=200;LMX=3
      GO TO 510
      IF(ISGMA.NE.1)GO TO 510
C---CHECKING OF DIOGONAL ELEMENTS OF F MATRIX ---
      DO 358 J=1,3
      SUMU=0.0
      DO 350 I=1,50
      SUMU=SUMU+(A(I,J))**2
      Y1=SUMU*S1
      SUMAL=0.0
      DO 352 I=51,100
      SUMAL=SUMAL+(A(I,J))**2
      Y2=SUMAL*S2
      SUMQ=0.0
      DO 354 I=101,150
      SUMQ=SUMQ+(A(I,J))**2
      Y3=SUMQ*S3
      SUMTH=0.0
      DO 356 I=151,200
      SUMTH=SUMTH+(A(I,J))**2
      Y4=SUMTH*S4
      DEF(J)=Y1+Y2+Y3+Y4
      CONTINUE
      WRITE(5,401)(DEF(J),J=1,3)
      FORMAT(5X,3E18.4)
      WRITE(10,402)(DEF(J),J=1,3)
      FORMAT(3X,'DEF'//(5X,3E18.4))
C---------------GENERATION OF D MATRIX ---------
      DO 501 I=1,50
      D(I,I)=S1
      DO 502 I=51,100
      D(I,I)=S2
      DO 503 I=101,150
```

```
D(I,I)=S3
DO 504 I=151,200
D(I,I)=S4
IF(ISGMA.NE.1)GO TO 263
-------TRANSPOSE OF A MATRIX -------------
DO 262 I=1,IMX
DO 262 J=1,LMX
AT(J,I)=A(I,J)
CONTINUE
------MULTIPLICATION OF AT AND D MATRIX ------
DO 264 I=1,LMX
DO 264 J=1,JMX
E(I,J)=AT(I,J)*D(J,J)
CONTINUE
----MULTIPLICATION OF E AND A MATRIX -----------
DO 266 I=1,LMX
DO 266 K=1,LMX
F(I,K)=0.0
DO 268 I=1,LMX
DO 268 K=1,LMX
DO 268 J=1,JMX
SUM1=E(I,J)*A(J,K)
F(I,K)=F(I,K)+SUM1
CONTINUE
WRITE(5,401)(F(I,I),I=1,3)
WRITE(10,403)(F(I,I),I=1,3)
FORMAT(3X,'DIOGONAL ELEMENTS OF F MATRIX'//(5X,3E18.4))
-------------INVERSION OF F MATRIX --------------
N=3;IA1=3
IUNIT=3;IFAIL=0
CALL F01AAF(F,IA1,N,UNIT,IUNIT,WKSPCE,IFAIL)
WRITE(5,*)IFAIL
DO 410 I=1,3
CRB(I)=SQRT(UNIT(I,I))
WRITE(5,401)(CRB(I),I=1,3)
IF(ISGMA.NE.1)GO TO 444
WRITE(10,404)(CRB(I),I=1,3)
FORMAT(3X,'CRAMER RAO BOUNDS FROM MEASUREMENT NOISE'//(5X,
1    3E18.4))
GO TO 446
WRITE(10,405)(CRB(I),I=1,3)
FORMAT(3X,'CRAMER RAO BOUNDS FROM ESTIMATED RESPONSE'
1    /(5X,3E18.4))
ISGMA=ISGMA+1
IF(ISGMA.GT.2)GO TO 132
GO TO 448
REWIND(3)
------ FORMATION OF MATRIX B ------
IF(NOISE.EQ.0)GO TO 133
REWIND(6)
DO 140 I=1,50
READ(6,61)U6,ALPHA6,Q6,THETA6
READ(3,*)U3,ALPHA3,Q3,THETA3
B(I,1)=(U6-U3)/U1
B(I+50,1)=ALPHA6-ALPHA3
B(I+100,1)=Q6-Q3
B(I+150,1)=THETA6-THETA3
REWIND(6)
GO TO 137
REWIND(2)
DO 161 I=1,50
READ(2,*)U2,ALPHA2,Q2,THETA2
READ(3,*)U3,ALPHA3,Q3,THETA3
B(I,1)=(U2-U3)/U1
B(I+50,1)=ALPHA2-ALPHA3
B(I+100,1)=Q2-Q3
```

```
B(I+150,1)=THETA2-THETA3
CONTINUE
------ SOLUTION OF LINEAR SIMULTANEOUS EQUATIONS ------
REWIND(2)
M=200;IA=200;IB=200;IDGT=6
NA=3
NB=1
CALL LLSQAR(A,B,M,NA,NB,IA,IB,IDGT,WKAREA,IER)
DO 145 I=1,3
X(I,1)=B(I,1)
----- UPDATING OF STORE PARAMETERS -----
XDELE=CDS+X(1,1)
CDS=XDELE
ZDELE=CLS+X(2,1)
CLS=ZDELE
MDELE=CMS+X(3,1)
CMS=MDELE
WRITE(10,142)ITER
FORMAT(10X,'ITERATION NO.',I3)
WRITE(5,*)CDS,CLS,CMS
WRITE(10,146)CDS,CLS,CMS
FORMAT(5X,'CDS=',F8.4,5X,'CLS=',F8.4,9X,'CMS=',F8.4)
ITER=ITER+1
WRITE(5,*)ITER
IF(ITER.GT.ITRMAX)GO TO 200
KCOUNT=2
REWIND(21)
REWIND(3)
GO TO 17
------ STORING OF ESTIMATED PARAMETERS ------
DD(1,IN)=CDS
DD(2,IN)=CLS
DD(3,IN)=CMS
WRITE(5,*)IN
IN=IN+1
IF(IN.GT.INMAX)GO TO 210
REWIND(1)
REWIND(2)
GO TO 300
WRITE(10,25)((DD(JN,IN),IN=1,INMAX),JN=1,3)
FORMAT(3X,'STORE PARAMETERS'/(5X,3E18.4))
IF(INMAX.EQ.1)GO TO 1000
CALCULATION OF MEAN AND SAMPLE STANDARD DEVIATION
DO 220 JN=1,3
SUM(JN)=0.0
DO 230 IN=1,INMAX
SUM(JN)=SUM(JN)+DD(JN,IN)
DDM(JN)=SUM(JN)/FLOAT(INMAX)
SMTN(JN)=0.0
DO 240 IN=1,INMAX
SMTN(JN)=SMTN(JN)+(DD(JN,IN)-DDM(JN))**2
CONTINUE
STDVN(JN)=SQRT(SMTN(JN)/FLOAT(INMAX-1))
CONTINUE
WRITE(10,26)(DDM(JN),JN=1,3)
FORMAT(5X,'MEAN VALUES OF ESTIMATED  STORE  PARAMETERS'
1   /(7X,3E20.4))
WRITE(10,27)(STDVN(JN),JN=1,3)
FORMAT(5X,'SAMPLE STANDARD DEVIATION OF PARAMETER ESTIMATES'/
1   (7X,3E20.4))
STOP
CALL LPSDOR
CALL LSVALR
CALL UERTST
CALL VSORTM
CALL GGUB
```

```
CALL MERFI
END
----------------------------------------------------------------
FUNCTION UDOT
FUNCTION UDOT(UX,ALPHAX,QX,THETAX)
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
UDOT=-G*THETAX*COS(THETA1)+(XU+XTU)*UX+XALPHA*ALPHAX+XQ*QX+XDELE
1    *DELE
RETURN
END
----------------------------------------------------------------
FUNCTION ALPDOT
FUNCTION ALPDOT(ALPHAX,UX,QX,THETAX)
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
ALPDOT=(-G*THETAX*SIN(THETA1)+ZU*UX+ZALPHA*ALPHAX+(ZQ+U1)*QX
1   +ZDELE*DELE)/(U1-ZALDOT)
RETURN
END
----------------------------------------------------------------
FUNCTION QDOT
FUNCTION QDOT(QX,UX,ALPHAX,THETAX)
REAL MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A1/XU,XTU,XALPHA,XQ,XDELE
COMMON/A2/MU,MTU,MALPHA,MTALPA,MALDOT,MQ,MDELE
COMMON/A3/U1,ZU,ZALPHA,ZQ,ZDELE,ZALDOT
COMMON/A4/G,DELE,THETA1
COMMON/A5/TIMINC
ALDOT=ALPDOT(ALPHAX,UX,QX,THETAX)
QDOT=(MU+MTU)*UX+(MALPHA+MTALPA)*ALPHAX+MALDOT*ALDOT+MQ*QX
1   +MDELE*DELE
RETURN
END
----------------------------------------------------------------
SUBROUTINE FOR RUNGE KUTTA FOURTH ORDER METHOD
SUBROUTINE RUNKUT(FUN,VAR1,VAR2,VAR3,VAR4,VAR5)
COMMON/A5/H
S1=FUN(VAR1,VAR2,VAR3,VAR4)
S2=FUN(VAR1+0.5*H*S1,VAR2,VAR3,VAR4)
S3=FUN(VAR1+0.5*H*S2,VAR2,VAR3,VAR4)
S4=FUN(VAR1+S3*H,VAR2,VAR3,VAR4)
S=(S1+S2*2.0+S3*2.0+S4)/6.0
VAR5=VAR1+S*H
RETURN
END
****************************************************************
```

TABLE - 1

Mass, moment of inertia, geometric characteristics and stability, control derivatives of test aircraft

m_a = 5000 kg ; I_{yy_a} = 30400 kg.m^2 ; S = 16.42 m^2 ;

b = 8.61 m ; $\bar{C}$ = 2.05 m ; U_1 = 137.5 m/sec

θ_1 = 0.0 rad ; g = 9.81 m/sec^2 ; ρ = 0.685 kg/m^3

Mach = 0.44 ; l_t = 4.2922 m

Configuration clean ; Position of C.G. : $X_{C.G.}$ = 25% of $\bar{C}$

Longitudinal stability and control derivatives

C_{D_u}	0.048	C_{m_u}	0.0
C_{D_1}	0.0375	C_{m_1}	0.0
C_{D_α}	0.41	C_{m_α}	-0.487
$C_{T_{X_u}}$	0.0	$C_{m_{\dot{\alpha}}}$	-1.8
$C_{T_{X_1}}$	0.0375	C_{m_q}	-4.4
C_{L_u}	0.0286	$C_{m_{\delta_e}}$	-0.67
C_{L_1}	0.47	$C_{m_{T_u}}$	0.0
C_{L_α}	3.9	$C_{m_{T_1}}$	0.0
$C_{L_{\dot{\alpha}}}$	0.8	$C_{m_{T_\alpha}}$	0.0
C_{L_q}	3.6	$C_{L_{\delta_e}}$	0.32

TABLE - 2

Arbitrary elevator input : time vs. deflection

Time, sec	δ_e, rad (Positive down)	Time, sec	δ_e, rad
0.0	0.0	1.1	0.1
0.1	0.015	1.2	0.094
0.2	0.028	1.3	0.076
0.3	0.038	1.4	0.04
0.4	0.046	1.5	0.004
0.5	0.051	1.6	-0.024
0.6	0.057	1.7	-0.033
0.7	0.065	1.8	-0.026
0.8	0.086	1.9	-0.022
0.9	0.092	2.0	0.0
1.0	0.098		

TABLE - 3

Standard deviation (σ_N) for various store locations corresponding to 5% noise level.

σ_N	Store Location			
	V1C1	V1C2	V2C1	V2C2
σ_u	0.0195	0.0115	0.0200	0.0140
σ_α	0.0002	0.0002	0.0003	0.0003
σ_q	0.0003	0.0003	0.0003	0.0003
σ_θ	0.0007	0.0005	0.0008	0.0006

TABLE - 4

True and Initial values of aircraft parameters for test case

Sl. No.	Aircraft Parameter	True Value	Initial Value
1.	$-X_u$	0.0074	0.0037
2.	X_α	1.2759	1.9140
3.	$X_{\delta e}$	0.0000	0.0000
4.	$-Z_u$	0.1498	0.0749
5.	$-Z_\alpha$	83.7317	41.8660
6.	$-Z_q$	0.6975	1.0463
7.	$-Z_{\delta e}$	6.8050	3.4020
8.	M_u	0.0000	0.0000
9.	$-M_\alpha$	3.4918	5.2377
10.	$-M_q$	0.3314	0.1657
11.	$-M_{\delta e}$	4.8040	7.2060

TABLE - 5

True and Initial values of store parameters and scale factor for various locations.

Sl. No.	Store Parameter	Store Location							
		V1C1		V1C2		V2C1		V2C2	
		True	Initial	True	Initial	True	Initial	True	Initial
1.	C_{D_S}	0.200	1.200	0.200	1.200	0.200	1.200	0.200	1.200
2.	C_{L_S}	0.800	4.800	0.520	3.120	0.750	4.500	0.700	4.200
3.	C_{M_S}	0.260	-1.040	-0.260	1.040	0.277	-1.108	-0.200	0.800
4.	Scale factor (S_{C_F})	0.8		0.93		0.875		0.915	

TABLE - 6a

Parameter estimates and CR bounds for location V2C2:Method 1.

Sl. No.	Para-meter	True Value	Noise level					
			1%		2%		5%	
			Estimated Value	σ CR	Estimated Value	σ CR	Estimated Value	σ CR
1.	$-X_u$	0.0074	0.0106	0.0019	0.0138	0.0038	0.0233	0.0096
2.	X_α	1.2759	1.2726	0.1836	1.2700	0.3669	1.2670	0.9139
3.	$-Z_u$	0.1498	0.1477	0.0057	0.1456	0.0115	0.1387	0.0290
4.	$-Z_\alpha$	83.7317	83.5911	0.5506	83.4493	1.0990	83.0160	2.7320
5.	$-Z_q$	0.6975	0.2192	0.3088	-0.2596	0.6192	-1.6998	1.5590
6.	M_u	0.0000	0.0000	0.0001	-0.0001	0.0002	-0.0001	0.0005
7.	$-M_\alpha$	3.4918	3.4819	0.0059	3.4721	0.0118	3.4429	0.0291
8.	$-M_q$	0.3314	0.3312	0.0047	0.3310	0.0094	0.3304	0.0234
9.	C_{D_S}	0.2000	0.1932	0.0229	0.1862	0.0459	0.1648	0.1149
10.	C_{L_S}	0.7000	0.6950	0.0121	0.6899	0.0241	0.6747	0.0603
11.	C_{M_S}	-0.2000	-0.2119	0.0274	-0.2239	0.0549	-0.2606	0.1372

TABLE - 6b

Cramer-Rao bounds of parameters for location V2C2 : Method 1

Sl. No.	Parameter	Noise level 1% σ_{CR}	1% $\bar{\sigma}_{CR}$	2% σ_{CR}	2% $\bar{\sigma}_{CR}$	5% σ_{CR}	5% $\bar{\sigma}_{CR}$
1	X_u	0.0019	0.0019	0.0038	0.0037	0.0096	0.0094
2	X_α	0.1836	0.1803	0.3669	0.3601	0.9139	0.8969
3	Z_u	0.0057	0.0066	0.0115	0.0132	0.0290	0.0332
4	Z_α	0.5506	0.5945	1.0990	1.1870	2.7320	2.9520
5	Z_q	0.3088	0.3490	0.6192	0.6997	1.5590	1.7620
6	M_u	0.0001	0.0001	0.0002	0.0002	0.0005	0.0005
7	M_α	0.0059	0.0066	0.0118	0.0131	0.0291	0.0323
8	M_q	0.0047	0.0052	0.0094	0.0104	0.0234	0.0259
9	C_{D_S}	0.0229	0.0225	0.0459	0.0450	0.1149	0.1127
10	C_{L_S}	0.0121	0.0131	0.0241	0.0262	0.0603	0.0655
11	C_{M_S}	0.0274	0.0294	0.0549	0.0588	0.1372	0.1470

TABLE - 7

Mean values and σ_S for location V2C2 : Method 1

Sl. No.	Parameter	True value	Noise level 1% Mean	1% σ_S	2% Mean	2% σ_S	5% Mean	5% σ_S
1.	$-X_u$	0.0074	0.0067	0.0020	0.0061	0.0040	0.0041	0.0100
2.	X_α	1.2759	1.2930	0.1495	1.3100	0.2990	1.3590	0.7484
3.	$-Z_u$	0.1498	0.1506	0.0079	0.1514	0.0157	0.1538	0.0394
4.	$-Z_\alpha$	83.7317	83.7500	0.5084	83.7600	1.0160	83.8100	2.5390
5.	$-Z_q$	0.6975	0.5877	0.3930	0.4757	0.7866	0.1274	1.9710
6.	M_u	0.0000	0.0000	0.0001	0.0001	0.0002	0.0001	0.0004
7.	$-M_\alpha$	3.4918	3.4890	0.0069	3.4870	0.0138	3.4800	0.0344
8.	$-M_q$	0.3314	0.3316	0.0059	0.3318	0.0118	0.3325	0.0294
9.	C_{D_S}	0.2000	0.1994	0.0178	0.1989	0.0356	0.1975	0.0889
10.	C_{L_S}	0.7000	0.6983	0.0092	0.6967	0.0184	0.6915	0.0461
11.	C_{M_S}	-0.2000	-0.2033	0.0221	-0.2066	0.0442	-0.2168	0.1107

TABLE - 8

Comparison of σ_S, σ_{CR} and $\bar{\sigma}_{CR}$ for location V2C2 : Method 1

Sl. No.	Parameter	Noise levels 1%		2%		5%	
		σ_S/σ_{CR}	$\sigma_S/\bar{\sigma}_{CR}$	σ_S/σ_{CR}	$\sigma_S/\bar{\sigma}_{CR}$	σ_S/σ_{CR}	$\sigma_S/\bar{\sigma}_{CR}$
1.	X_u	1.0526	1.0526	1.0526	1.0811	1.0417	1.0638
2.	X_α	0.8143	0.8292	0.8149	0.8303	0.8189	0.8344
3.	Z_u	1.3860	1.1970	1.3652	1.1894	1.3586	1.1867
4.	Z_α	0.9234	0.8552	0.9245	0.8559	0.9294	0.8601
5.	Z_q	1.2727	1.1261	1.2703	1.1242	1.2643	1.1186
6.	M_u	1.0000	1.0000	1.0000	1.0000	0.8000	0.8000
7.	M_α	1.1695	1.0455	1.1695	1.0534	1.1821	1.0650
8.	M_q	1.2553	1.1346	1.2553	1.1346	1.2564	1.1351
9.	C_{D_S}	0.7773	0.7911	0.7756	0.7911	0.7737	0.7888
10.	C_{L_S}	0.7603	0.7023	0.7635	0.7023	0.7645	0.7038
11.	C_{M_S}	0.8066	0.7517	0.8051	0.7617	0.8069	0.7531

TABLE - 9a

Estimated values and CR bounds of aircraft parameters from a single measured response; input : elevator deflection

Sl. No.	Parameter	True value	Initial value	No-noise Estimated value	Noise level 1% Estimated value	Noise level 1% $\bar{\sigma}_{CR}$	Noise level 2% Estimated value	Noise level 2% $\bar{\sigma}_{CR}$
1.	$-X_u$	0.0074	0.0037	0.0074	0.0075	0.0008	0.0068	0.0016
2.	X_α	1.2759	1.9140	1.2759	1.3038	0.0452	1.3392	0.0905
3.	X_{δ_e}	0.0000	0.0000	0.0000	0.0355	0.0620	0.0479	0.1240
4.	$-Z_u$	0.1498	0.0749	0.1498	0.1517	0.0060	0.1500	0.0121
5.	$-Z_\alpha$	83.7317	41.8660	83.7317	83.1897	0.2515	83.6087	0.5042
6.	$-Z_q$	0.6975	1.0463	0.6975	0.5372	0.1973	0.7177	0.3948
7.	$-Z_{\delta_e}$	6.8050	3.4020	6.8050	6.7961	—	6.7938	—
8.	M_u	0.0000	0.0000	0.0000	0.0004	0.0001	0.0007	0.0002
9.	$-M_\alpha$	3.4918	5.2377	3.4918	3.4955	0.0046	3.5031	0.0092
10.	$-M_q$	0.3314	0.1657	0.3314	0.3353	0.0027	0.3330	0.0054
11.	$-M_{\delta_e}$	4.8040	7.2060	4.8040	4.7977	0.0059	4.7961	0.0118

TABLE - 9b

Estimated values and CR bounds of aircraft parameters from a single measured response; input : elevator deflection.

Sl. No.	Para-meter	True value	Initial value	Noise level 5% Estimated value	5% $\bar{\sigma}_{CR}$	10% Estimated value	10% $\bar{\sigma}_{CR}$
1.	$-X_u$	0.0074	0.0037	0.0043	0.0039	-0.0005	0.0078
2.	X_α	1.2759	1.9140	1.4476	0.2280	1.6345	0.4608
3.	X_{δ_e}	0.0000	0.0000	0.0751	0.3096	0.0862	0.6189
4.	$-Z_u$	0.1498	0.0749	0.1448	0.0303	0.1343	0.0603
5.	$-Z_\alpha$	83.7317	41.8660	84.8728	1.2700	87.0003	2.5700
6.	$-Z_q$	0.6075	1.0463	1.2581	0.9877	2.1567	1.9770
7.	$-Z_{\delta_e}$	6.8050	3.4020	6.7870	-	6.7748	-
8.	$-M_u$	0.0000	0.0000	0.0015	0.0006	0.0029	0.0012
9.	$-M_\alpha$	3.4918	5.2377	3.5262	0.0234	3.5658	0.0475
10.	$-M_q$	0.3314	0.1657	0.3258	0.0136	0.3133	0.0275
11.	$-M_{\delta_e}$	4.8040	7.2060	4.7913	0.0296	4.7827	0.0592

TABLE - 10

Mean values of aircraft parameters from 20 different measured responses for an identical elevator input.

Sl. No.	Parameter	True Value	Initial Value	Mean Value of estimated parameters			
				Noise level			
				1%	2%	5%	10%
1.	$-X_u$	0.0074	0.0037	0.0076	0.0077	0.0080	0.0082
2.	X_α	1.2759	1.9140	1.2758	1.2751	1.2580	1.2340
3.	$X_{\delta e}$	0.0000	0.0000	0.0120	0.0184	0.0365	0.0537
4.	$-Z_u$	0.1498	0.0749	0.1496	0.1495	0.1492	0.1489
5.	$-Z_\alpha$	83.7317	41.8660	83.7250	83.7106	83.6623	83.5908
6.	$-Z_q$	0.6975	1.0463	0.7315	0.7954	0.9992	1.2950
7.	$-Z_{\delta e}$	6.8050	3.4020	6.8052	6.8060	6.8065	6.8096
8.	M_u	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
9.	$-M_\alpha$	3.4918	5.2377	3.4930	3.4952	3.5010	3.5120
10.	$-M_q$	0.3314	0.1657	0.3312	0.3309	0.3303	0.3291
11.	$-M_{\delta e}$	4.8040	7.2060	4.8042	4.8046	4.8051	4.8068

TABLE - 11

Comparison of estimates of Case 1 and Case 2 for location V1C1

Sl. No.	Store Parameter	True value	Initial value	Estimated value and σ_{CR}^{*}					
				Noise level					
				1%		2%		5%	
				Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
1.	C_{D_S}	0.2000	1.2000	0.1919	0.2020	0.1840	0.2033	0.1615	0.2058
				(0.0321)	(0.0035)	(0.0641)	(0.0069)	(0.1595)	(0.0169)
2.	C_{L_S}	0.8000	4.8000	0.7933	0.8246	0.7866	0.8191	0.7666	0.8012
				(0.0136)	(0.0023)	(0.0273)	(0.0044)	(0.0681)	(0.0106)
3.	C_{M_S}	0.2600	-1.0400	0.2757	0.2118	0.2915	0.2269	0.3386	0.2746
				(0.0305)	(0.0059)	(0.0610)	(0.0113)	(0.1525)	(0.0274)

* σ_{CR} values given in brackets ( ).

TABLE - 12

Comparison of estimates of Case 1 and Case 2 for location V1C2

Sl. No.	Store Para-meter	True Value	Initial Value	Estimated Values and σ_{CR}^{*}					
				Noise level					
				1%		2%		5%	
				Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
1	C_{D_S}	0.2000	1.2000	0.1949 (0.0181)	0.2044 (0.0019)	0.1897 (0.0363)	0.2097 (0.0038)	0.1734 (0.0909)	0.2277 (0.0094)
2	C_{L_S}	0.5200	3.1200	0.5157 (0.0096)	0.5060 (0.0016)	0.5114 (0.0191)	0.5119 (0.0031)	0.4983 (0.0478)	0.5310 (0.0076)
3	C_{M_S}	-0.2600	1.0400	-0.2683 (0.0203)	-0.2928 (0.0027)	-0.2767 (0.0406)	-0.2827 (0.0054)	-0.3021 (0.1016)	-0.2489 (0.0131)

* σ_{CR} values given in brackets.

TABLE - 13

Comparison of estimates of Case 1 and Case 2 for location V2C1

Sl. No.	Store Parameter	True Value	Initial Value	Estimated Values and σ_{CR}^{*}					
				Noise Level					
				1%		2%		5%	
				Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
1	C_{D_S}	0.2000	1.2000	0.1924	0.2019	0.1850	0.2031	0.1638	0.2055
				(0.0300)	(0.0033)	(0.0599)	(0.0064)	(0.1491)	(0.0158)
2	C_{L_S}	0.7500	4.5000	0.7435	0.7728	0.7371	0.7678	0.7179	0.7513
				(0.0129)	(0.0021)	(0.0257)	(0.0041)	(0.0642)	(0.0099)
3	C_{M_S}	0.2770	-1.1080	0.2900	0.2329	0.3030	0.2471	0.3423	0.2916
				(0.0318)	(0.0058)	(0.0636)	(0.0111)	(0.1589)	(0.0268)

* σ_{CR} Values given in brackets

TABLE - 14

Comparison of estimates of Case 1 and Case 2 for location V2C2

Sl. No.	Store Parameter	True value	Initial value	Estimated value and σ_{CR}*					
				Noise level					
				1%		2%		5%	
				Case 1	Case 2	Case 1	Case 2	Case 1	Case
1	C_{D_S}	0.2000	1.2000	0.1932	0.2052	0.1862	0.2114	0.1648	0.23
				(0.0229)	(0.0023)	(0.0459)	(0.0046)	(0.1149)	(0.01
2	C_{L_S}	0.7000	4.2000	0.6950	0.6826	0.6899	0.6903	0.6747	0.714
				(0.0121)	(0.0019)	(0.0241)	(0.0038)	(0.0603)	(0.00
3	C_{M_S}	-0.2000	0.8000	-0.2119	-0.2396	-0.2239	-0.2247	-0.2606	-0.17
				(0.0274)	(0.0033)	(0.0549)	(0.0065)	(0.1372)	(0.01

*σ_{CR} values given in brackets

TABLE - 15

Comparison of σ_S and σ_{CR} of store parameters for location V1C2:Method 2.

Sl. No.	Store Parameter	True value	Noise level								
			1%			2%			5%		
			Case 2 Estimated value & σ^*_{CR}	Case3 Mean & σ^*_S	$\frac{\sigma_S}{\sigma_{CR}}$	Case 2 Estimated value & σ_{CR}	Case 3 Mean & σ_S	$\frac{\sigma_S}{\sigma_{CR}}$	Case 2 Estimated value & σ_{CR}	Case 3 Mean & σ_S	$\frac{\sigma_S}{\sigma_{CR}}$
1	C_{D_S}	0.2000	0.2044 (0.0019)	0.2009 (0.0021)	1.1053	0.2097 (0.0038)	0.2025 (0.0043)	1.1316	0.2277 (0.0094)	0.2086 (0.0112)	1.1915
2.	C_{L_S}	0.5200	0.5060 (0.0016)	0.5046 (0.0013)	0.8125	0.5119 (0.0031)	0.5092 (0.0025)	0.8065	0.5310 (0.0076)	0.5239 (0.0064)	0.8421
3	C_{M_S}	-0.2600	-0.2928 (0.0027)	-0.2955 (0.0020)	0.7407	-0.2827 (0.0054)	-0.2881 (0.0040)	0.7407	-0.2489 (0.0131)	-0.2630 (0.0101)	0.7710

*Values given in brackets

TABLE - 16

Comparison of estimates of cases 2,4 and 5 for location V2C1:Method 2

Sl. No.	Store Para-meter	True Value	Estimated Value and σ^*_{CR}								
			Noise level								
			1%			5%			10%		
			Case 2	Case 4	Case 5	Case 2	Case 4	Case 5	Case 2	Case 4	Case5
1	C_{D_S}	0.2000	0.2019 (0.0033)	0.2063 (0.0032)	0.2056 (0.0032)	0.2055 (0.0158)	0.2289 (0.0160)	0.2281 (0.0160)	0.1991 (0.0312)	0.2562 (0.0320)	0.2561 (0.0320)
2	C_{L_S}	0.7500	0.7728 (0.0021)	0.7516 (0.0018)	0.7509 (0.0018)	0.7513 (0.0099)	0.7551 (0.0091)	0.7543 (0.0092)	0.7238 (0.0198)	0.7589 (0.0182)	0.7586 (0.0184)
3	C_{M_S}	0.2770	0.2329 (0.0058)	0.2779 (0.0049)	0.2769 (0.0050)	0.2916 (0.0268)	0.2799 (0.0246)	0.2763 (0.0248)	0.3612 (0.0535)	0.2785 (0.0491)	0.2756 (0.0496)

σ_{CR} values given in brackets.

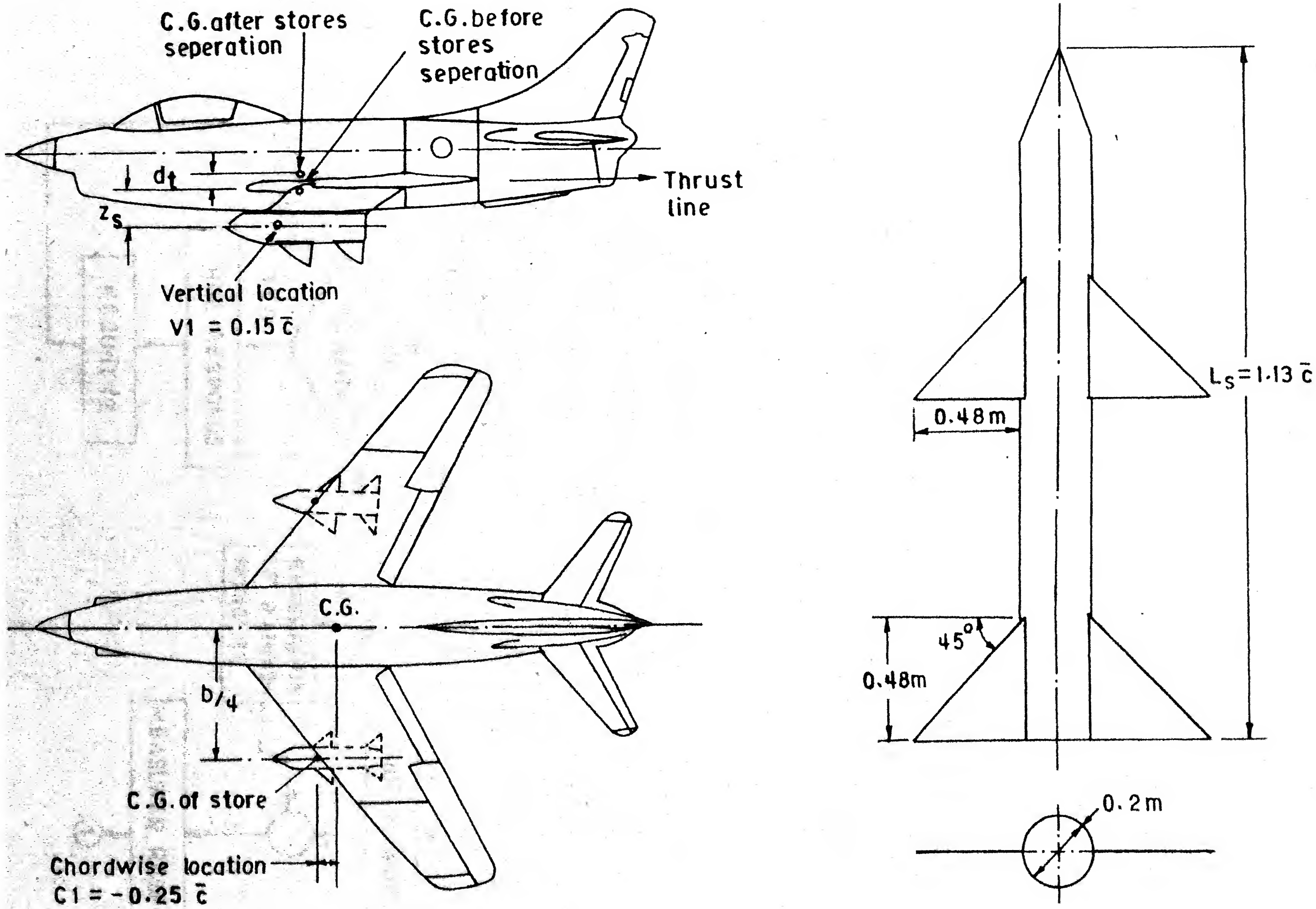


FIG.1 – AIRPLANE-STORE CONFIGURATION AND STORE GEOMETRY FOR THE TEST CASE

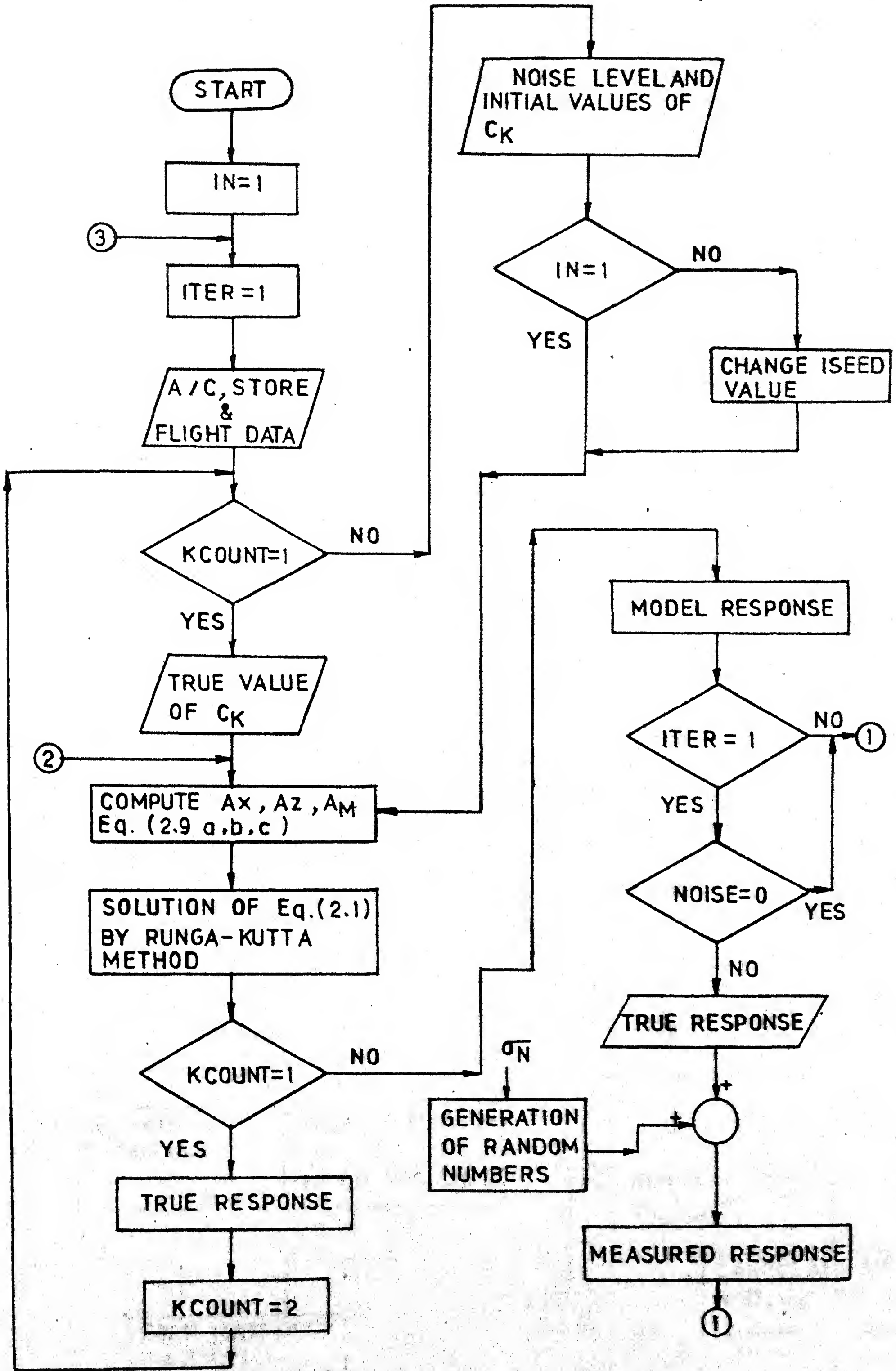


FIG. 2 - FLOW CHART OF COMPUTER CODE FOR ESTIMATION OF STORE PARAMETERS THROUGH GN METHOD

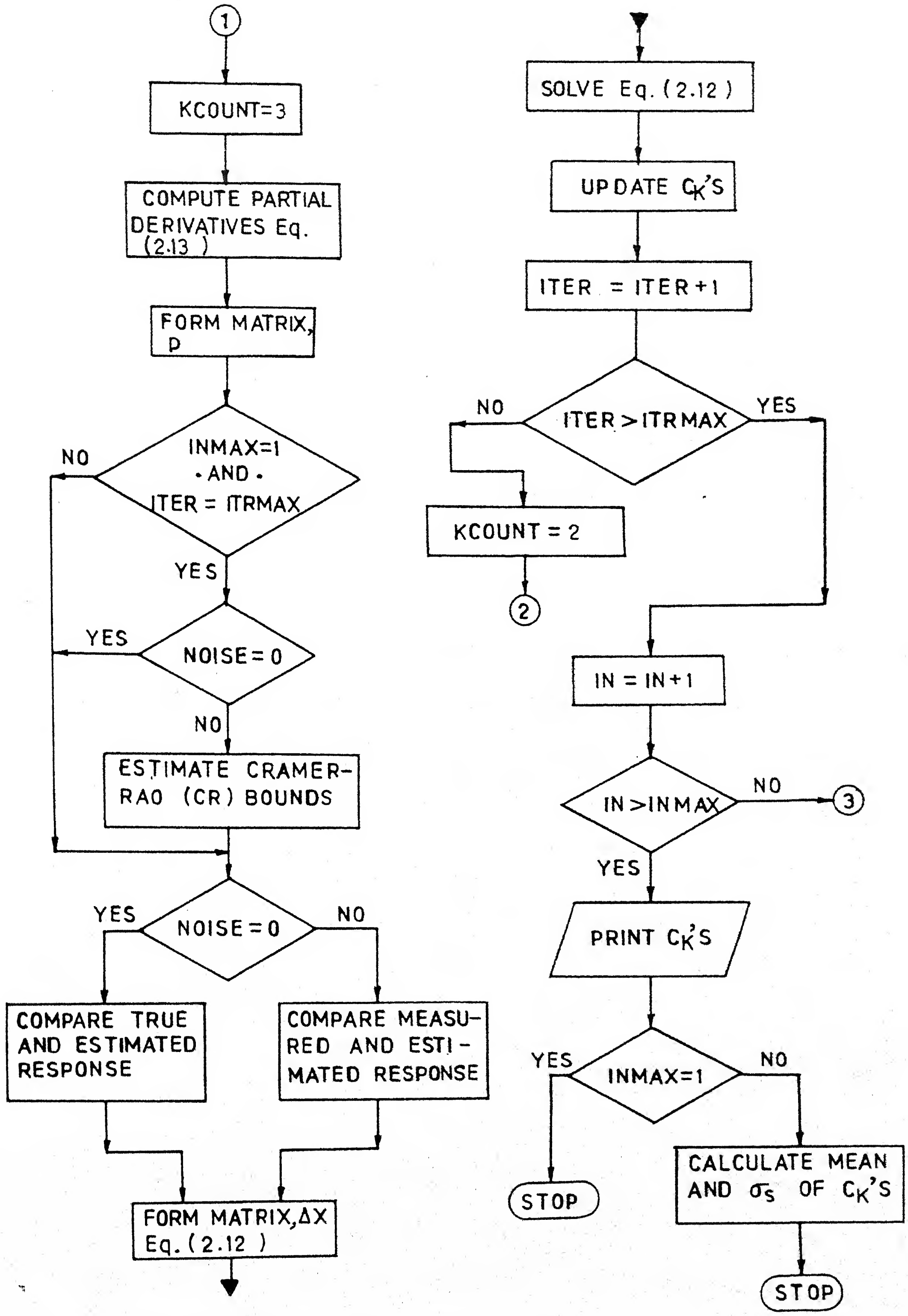


FIG. 2 - CONCLUDED

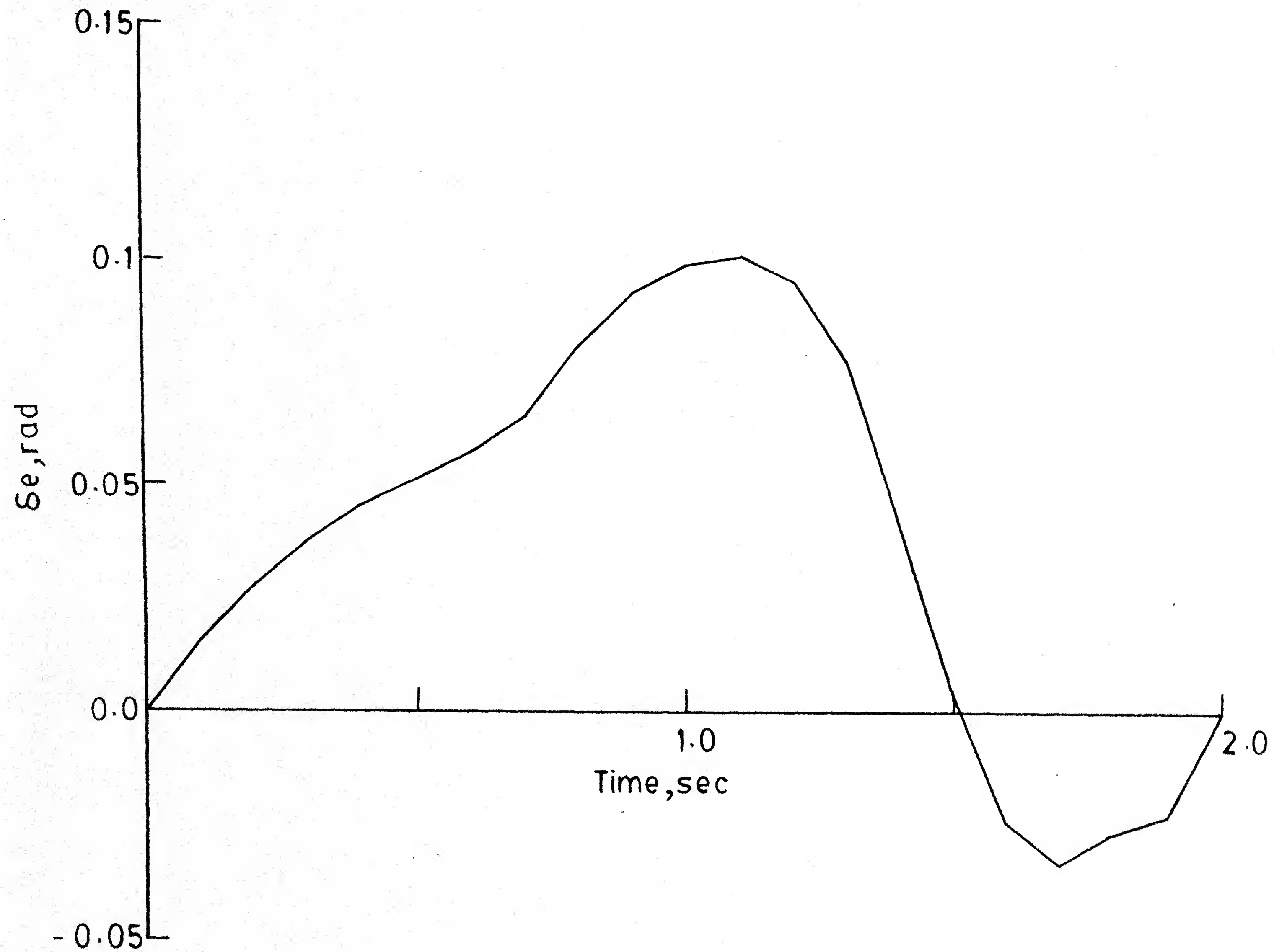


FIG. 3 – TIME HISTORY OF ELEVATOR INPUT

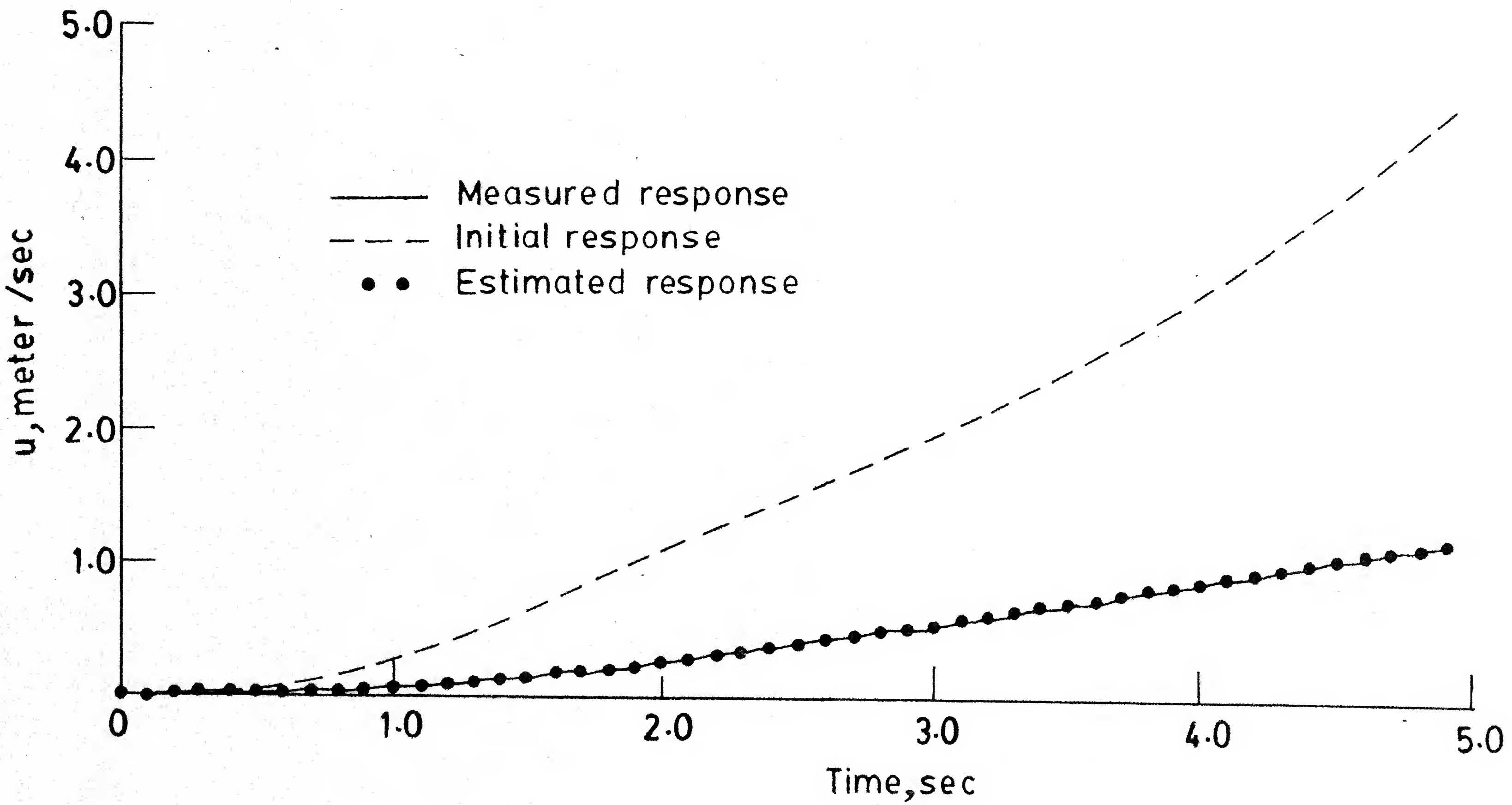


FIG. 4 – COMPARISON OF MEASURED, INITIAL AND ESTIMATED RESPONSE;
Location: V1 C1 ; Case: 1; Noise: Zero.

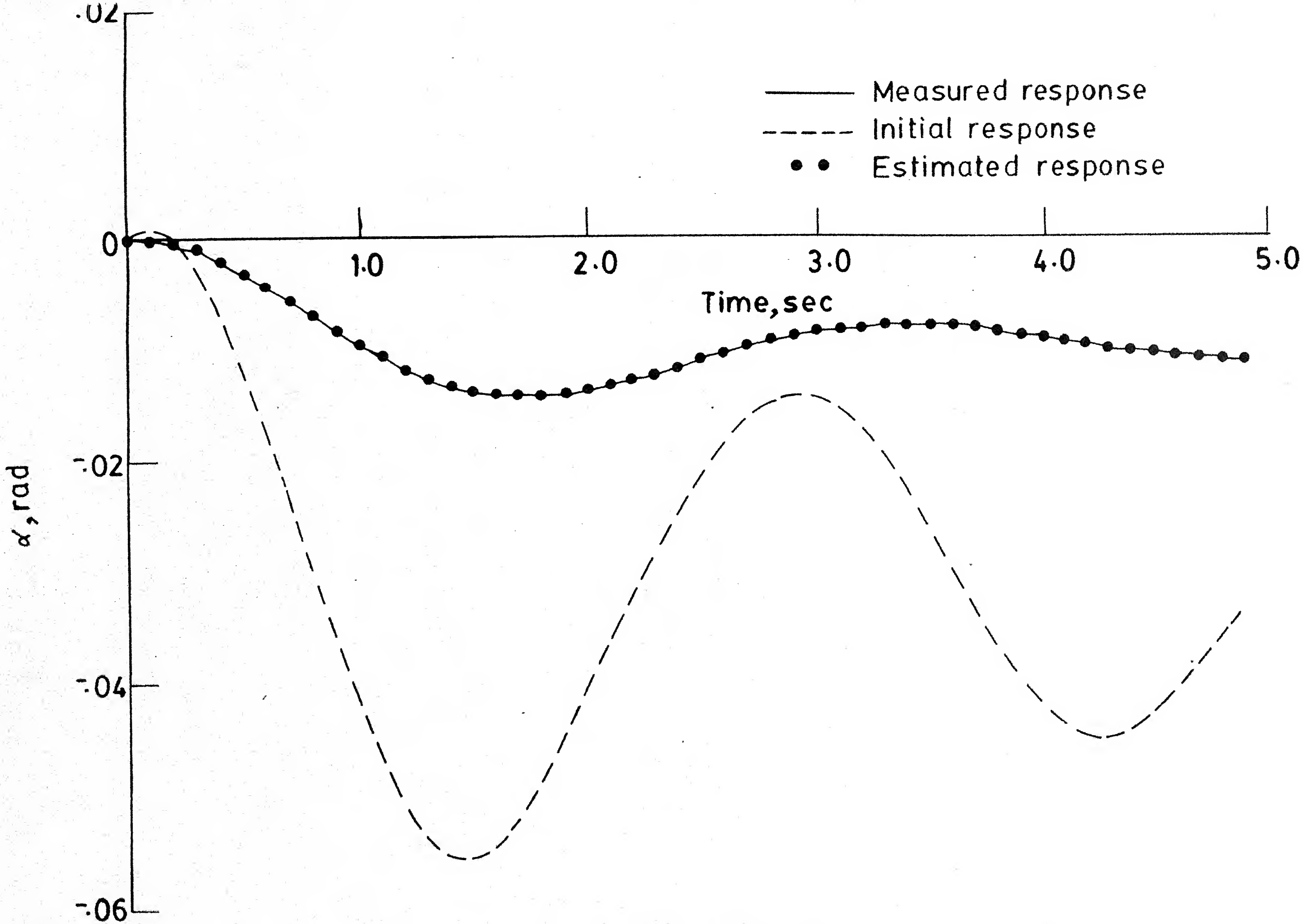


FIG. 4-Continued

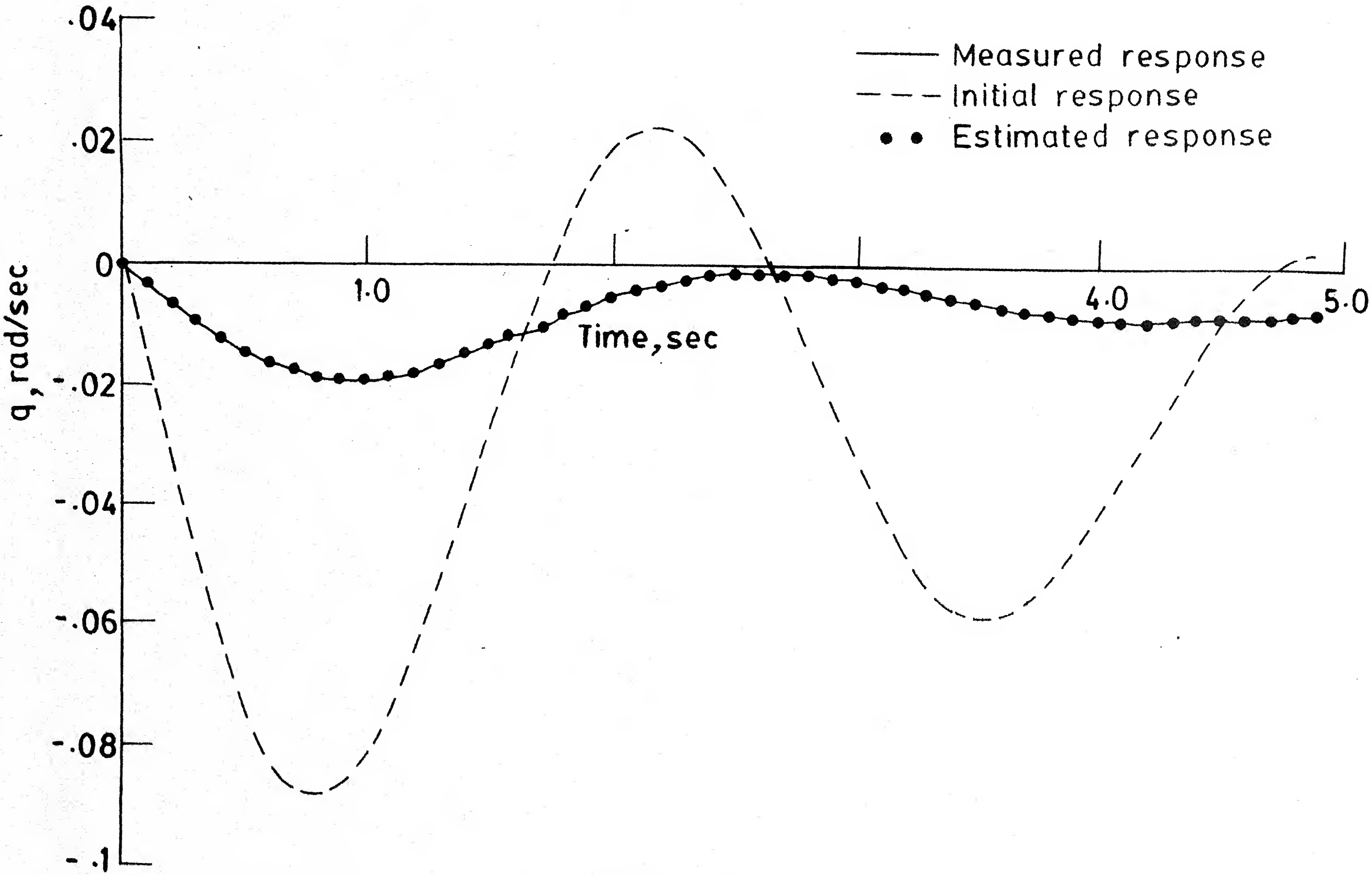


FIG. 4 – Continued

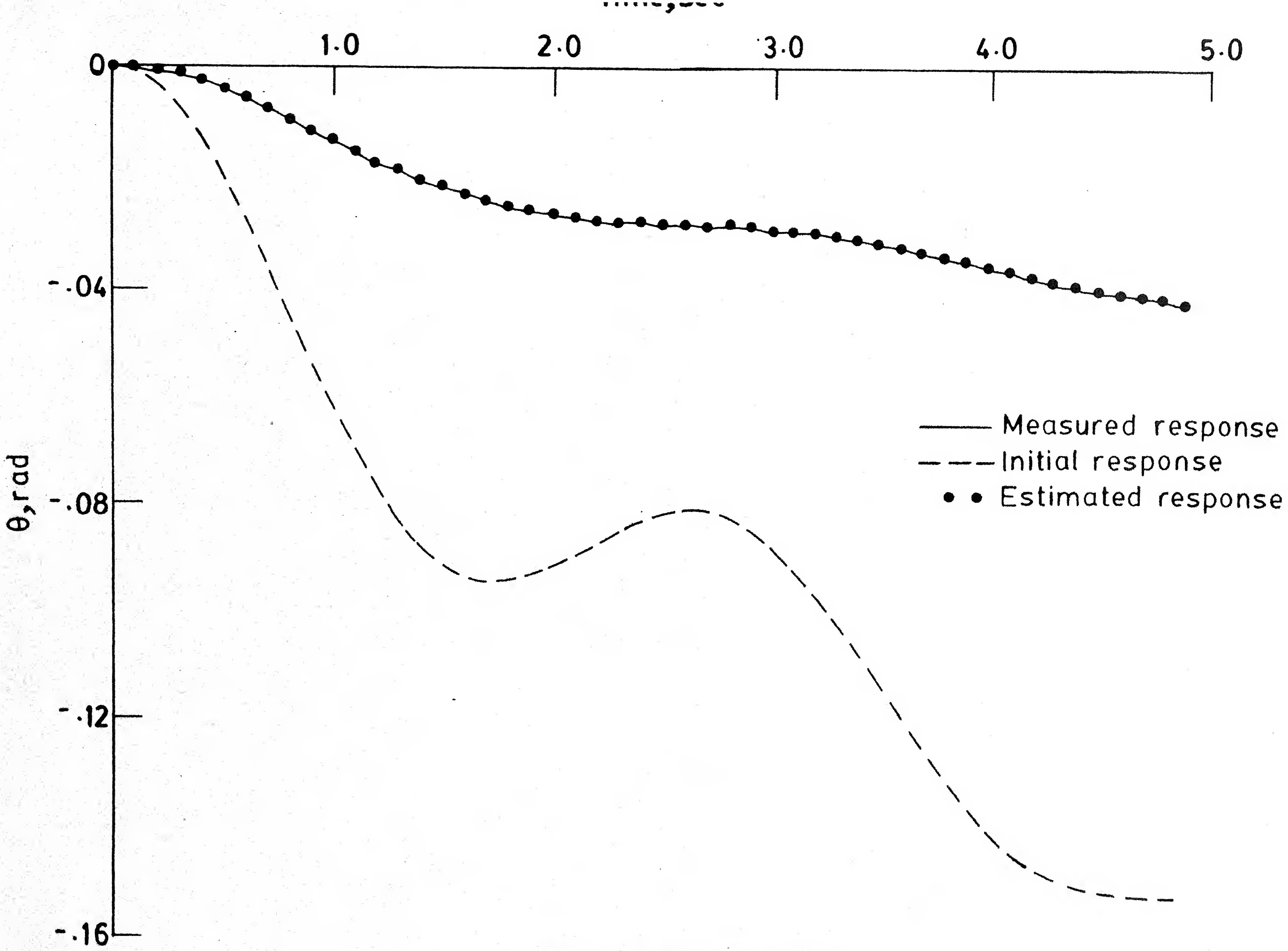


FIG. 4 — CONCLUDED

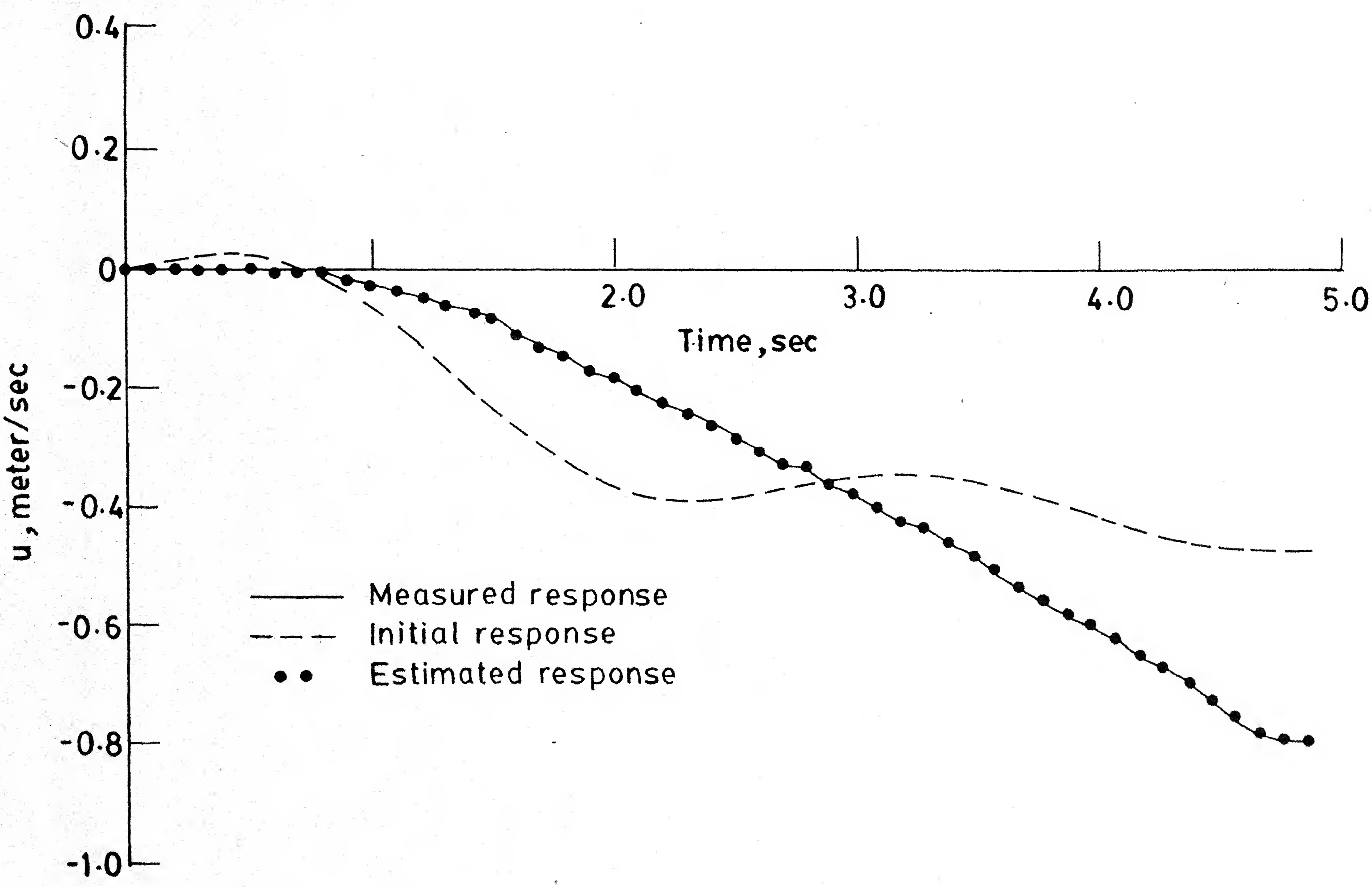
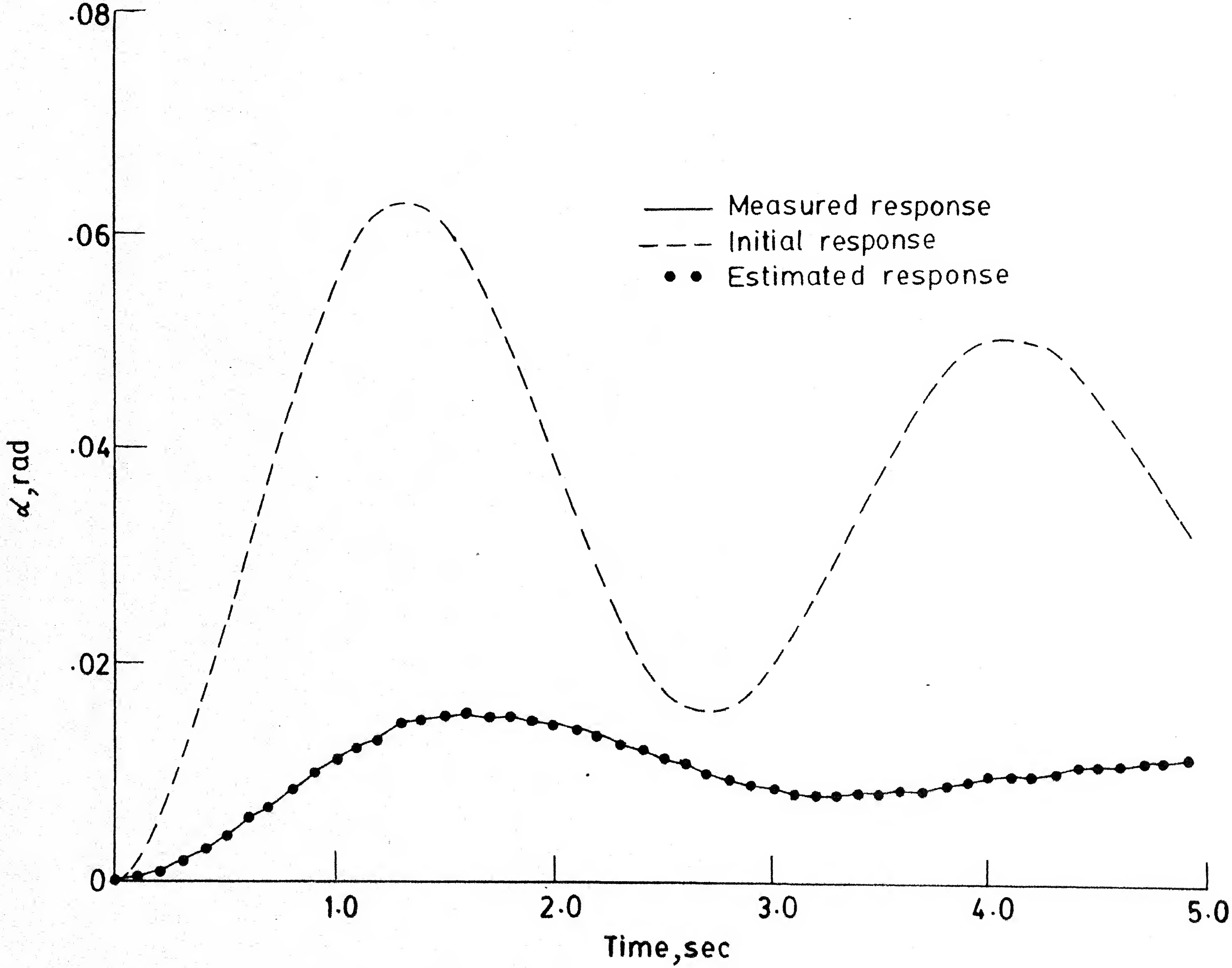


FIG. 5 – COMPARISON OF MEASURED, INITIAL AND ESTIMATED RESPONSE;
Location : V2C2 ; Case : 1; Noise 2 %



.08
.06
.04
.02
0
α, rad
1.0
2.0
3.0
4.0
5.0
Time, sec
Measured response
Initial response
Estimated response

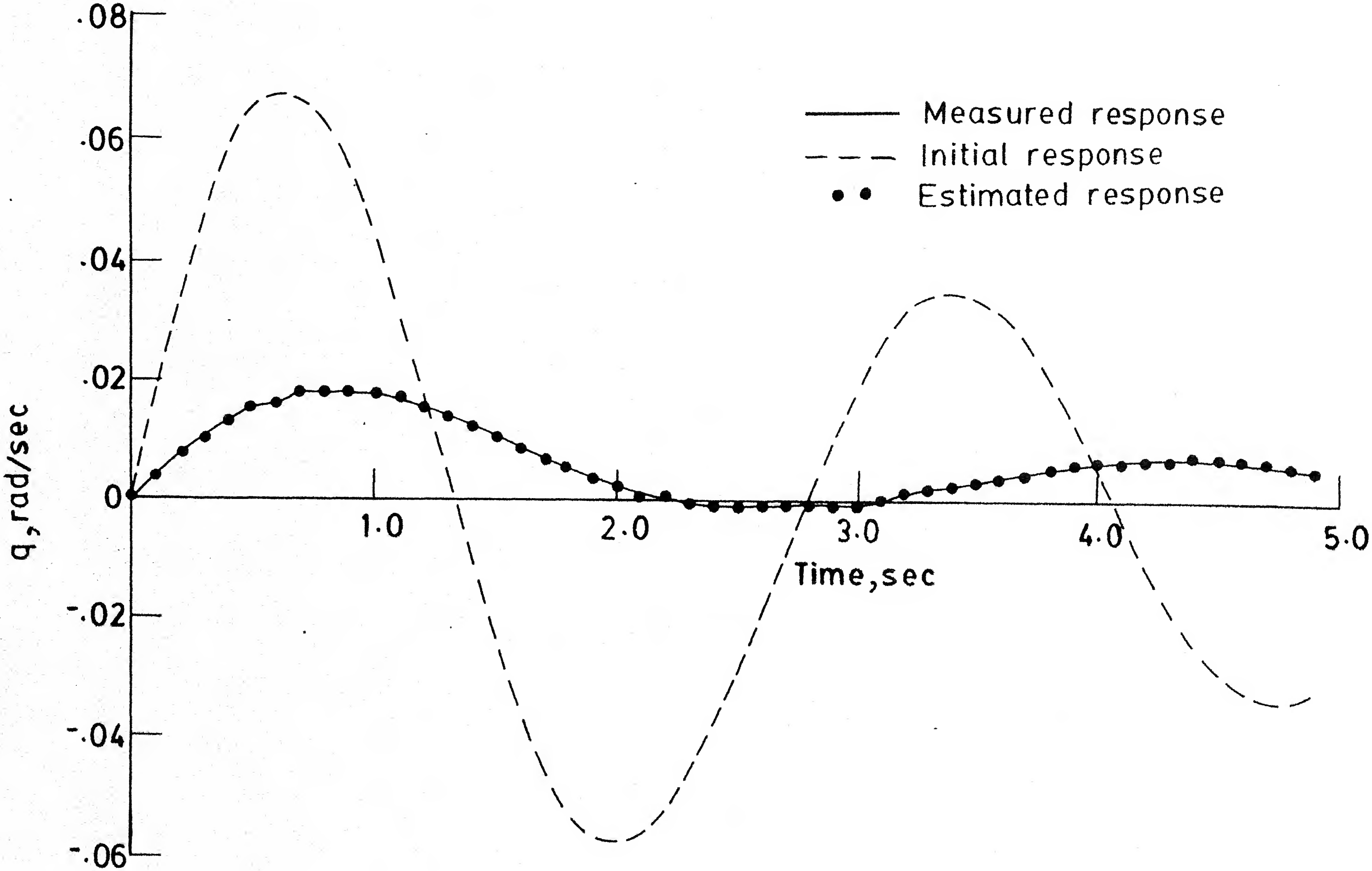


FIG.5 - Continued

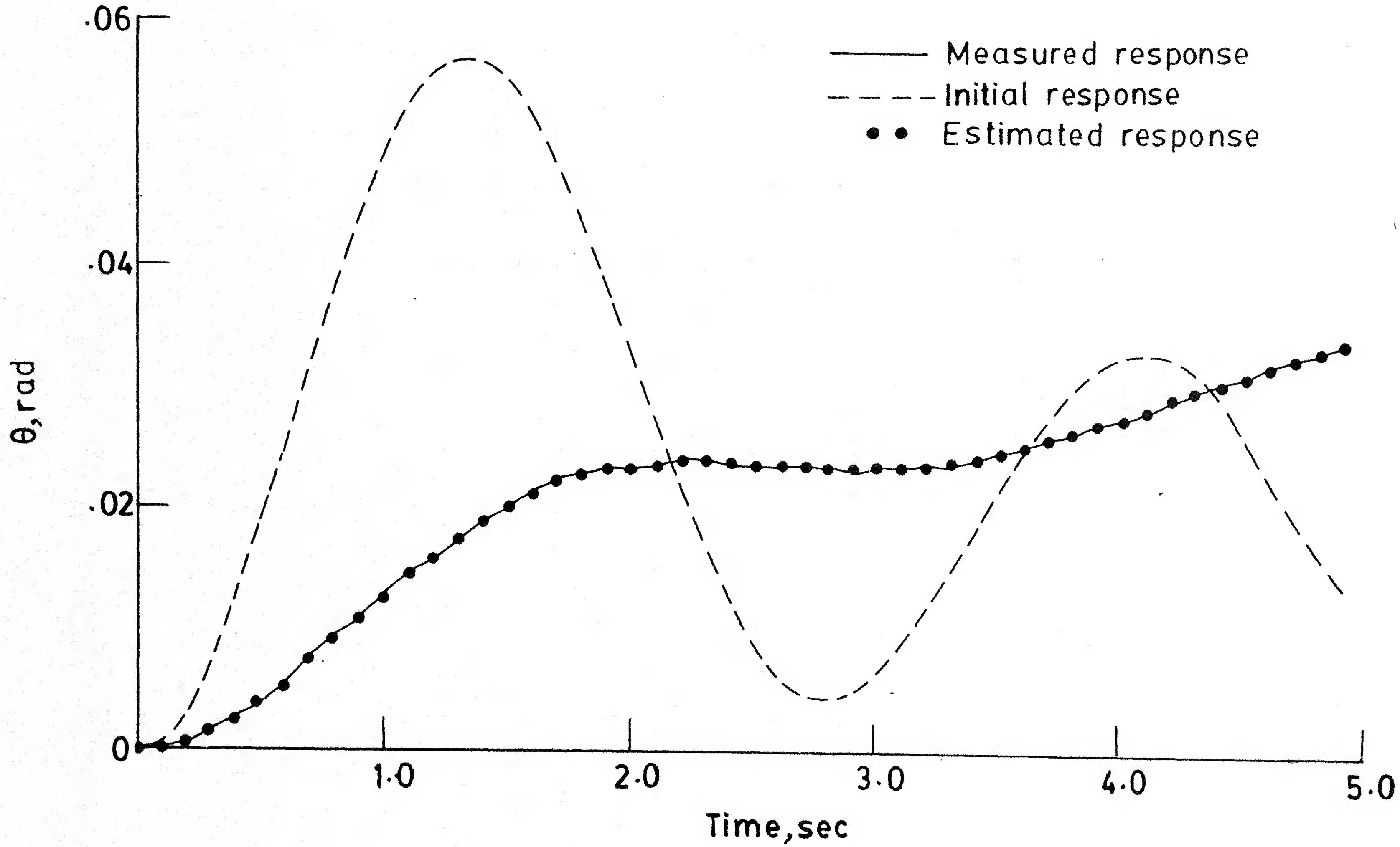


FIG. 5 — CONCLUDED

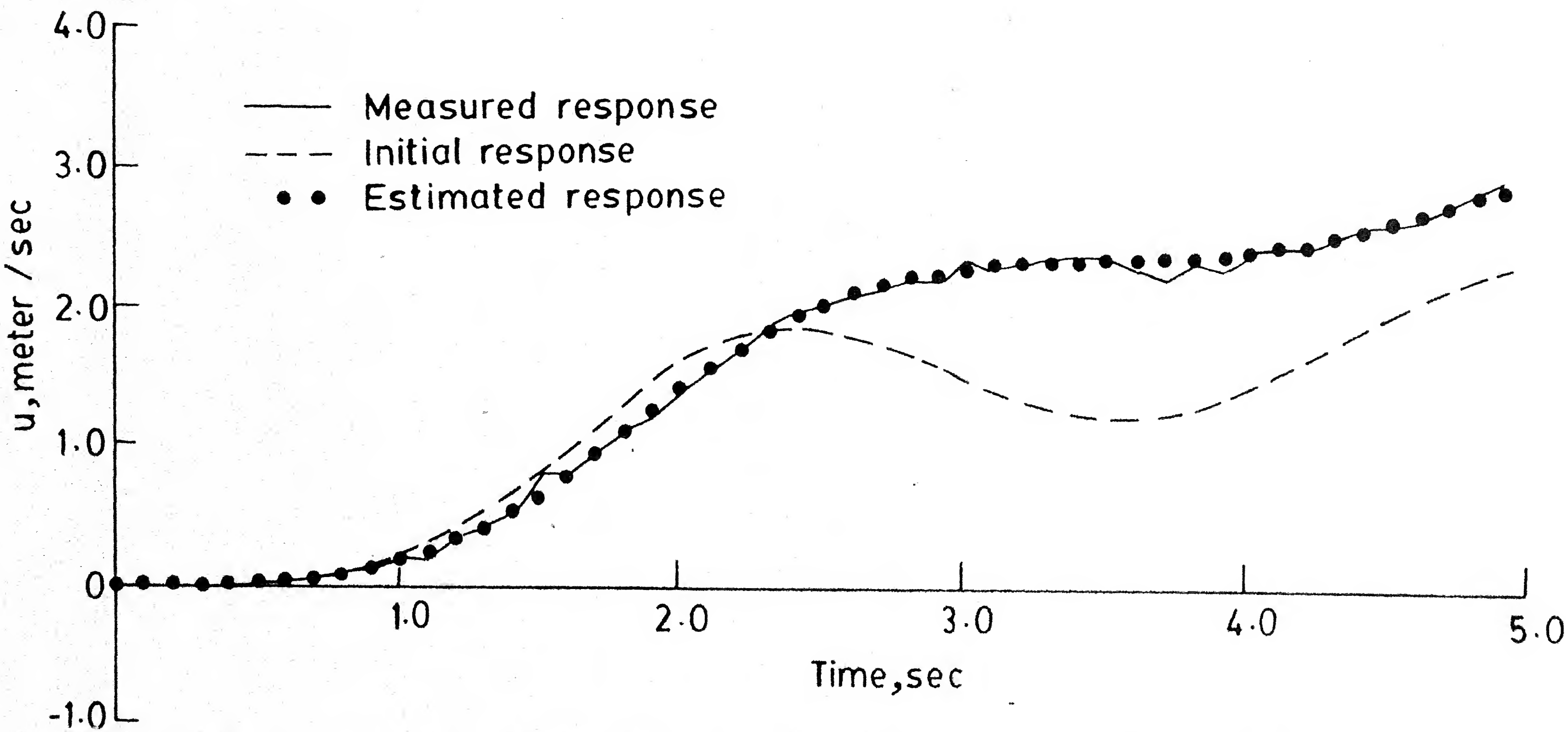


FIG. 6 — COMPARISON OF MEASURED, INITIAL AND ESTIMATED RESPONSE.
Noise: 5 % ; Input : Elevator deflection.

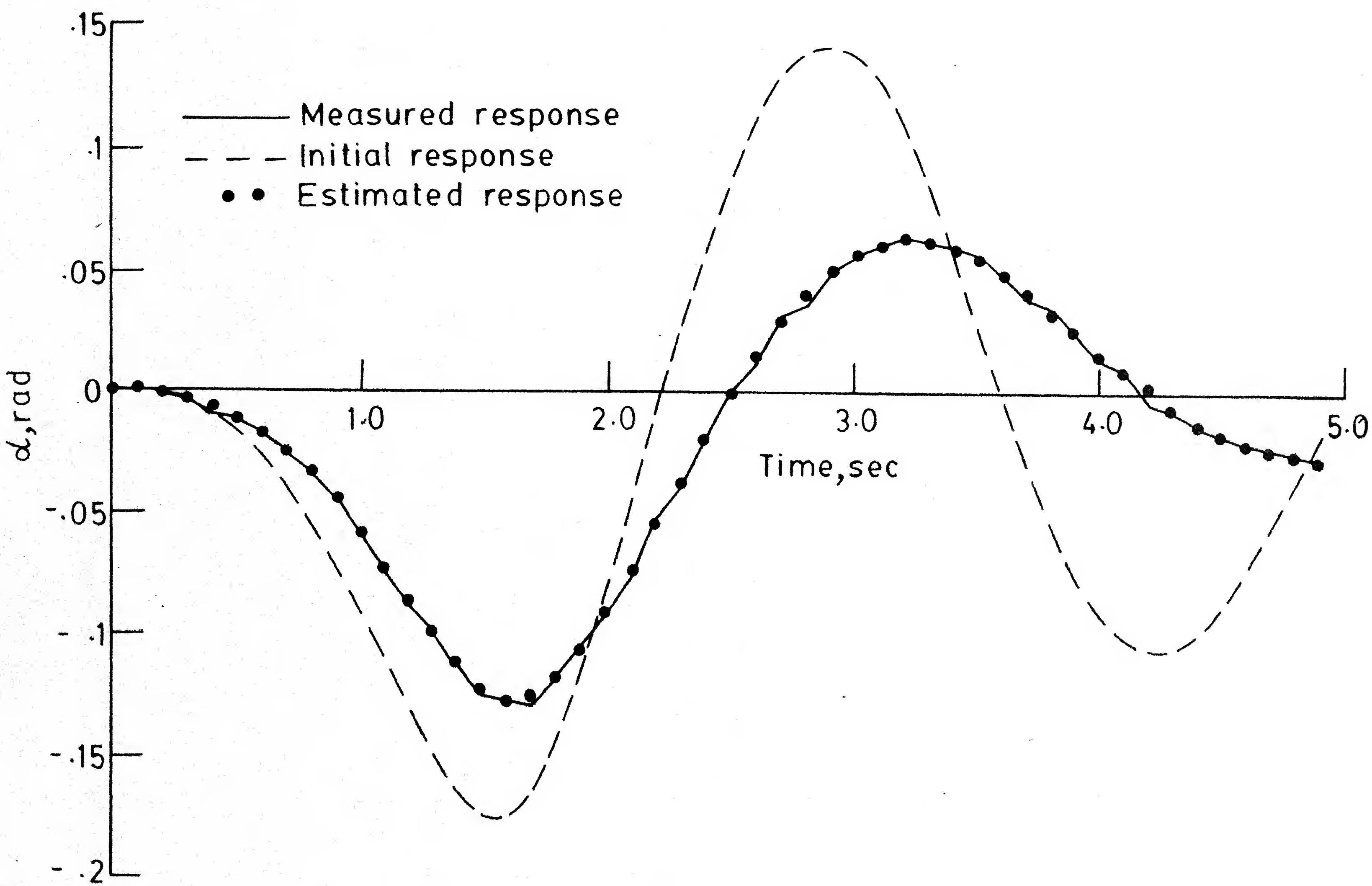


FIG. 6 – Continued

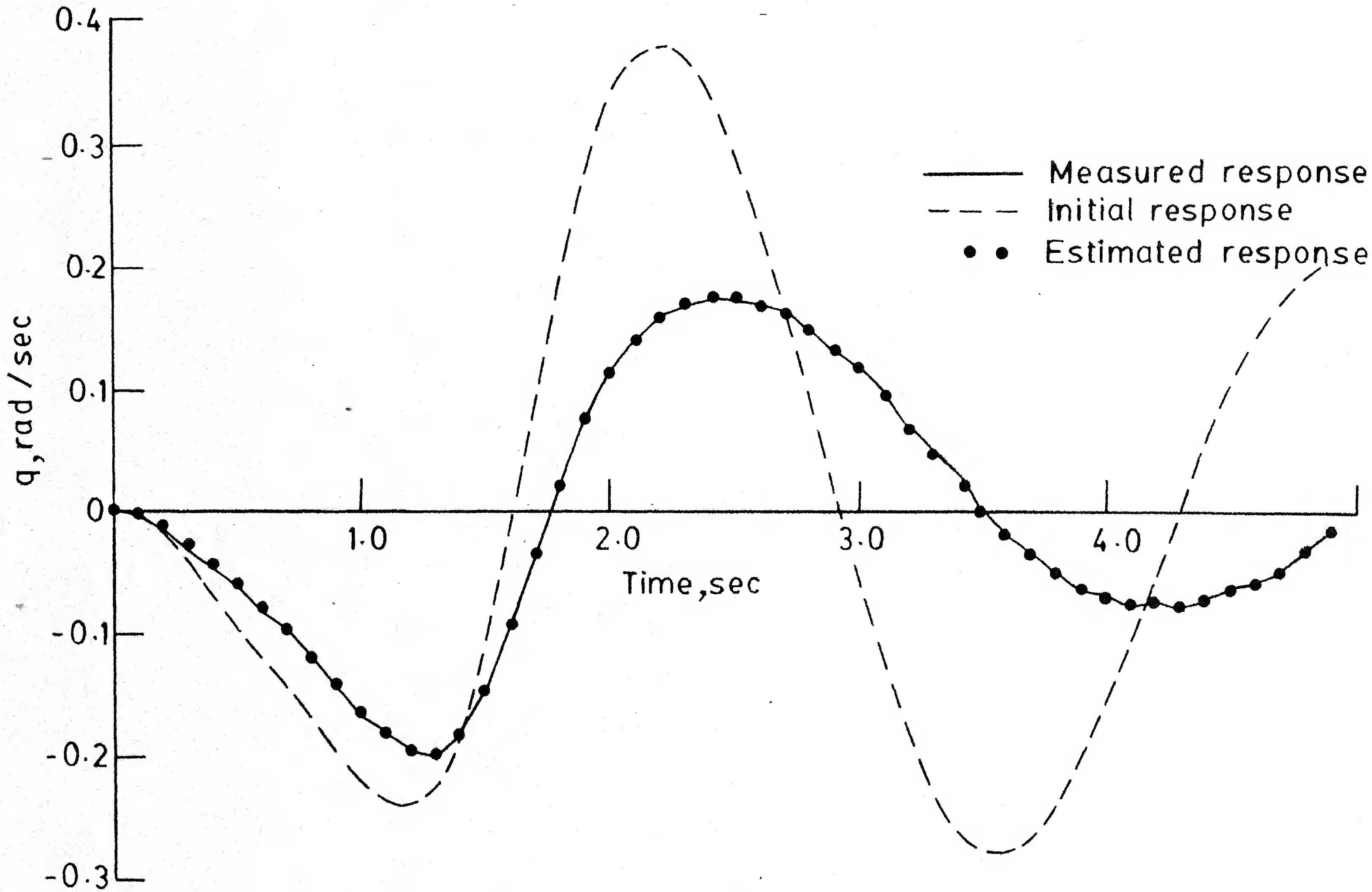


FIG.6–Continued

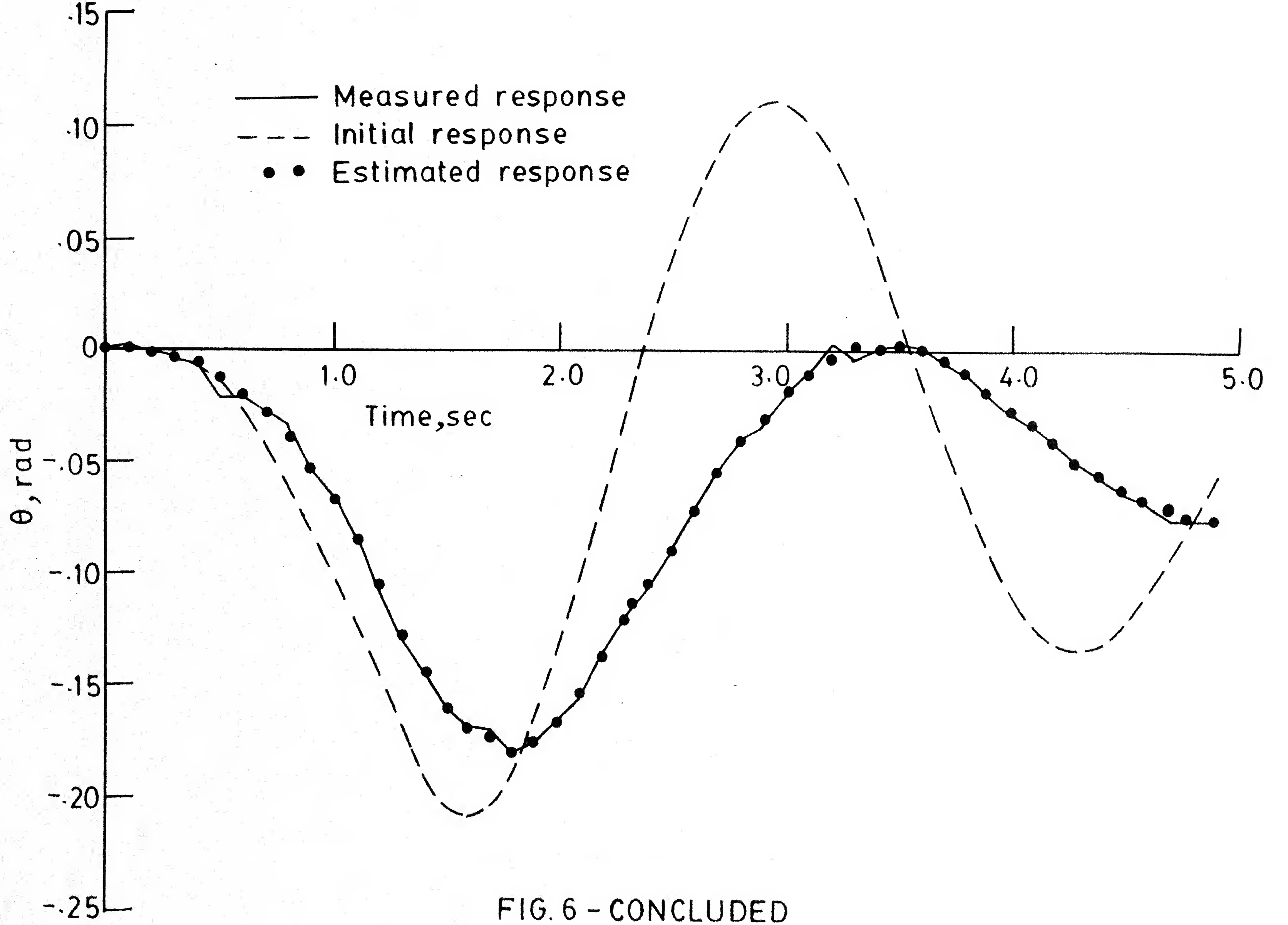


FIG. 6 - CONCLUDED

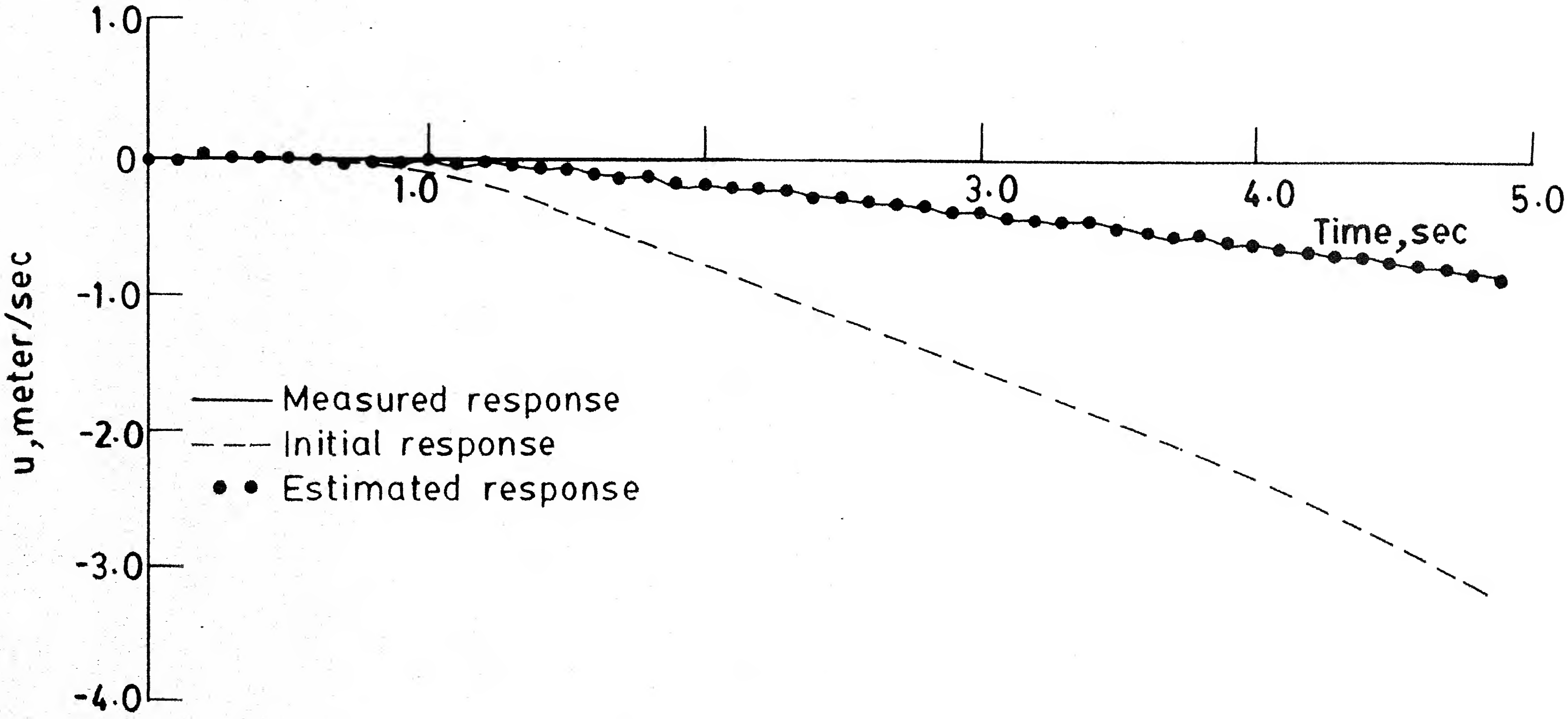


FIG. 7— COMPARISON OF MEASURED, INITIAL AND ESTIMATED RESPONSE
Location: V2 C2 ; Case: 2 ; Noise: 5 % .

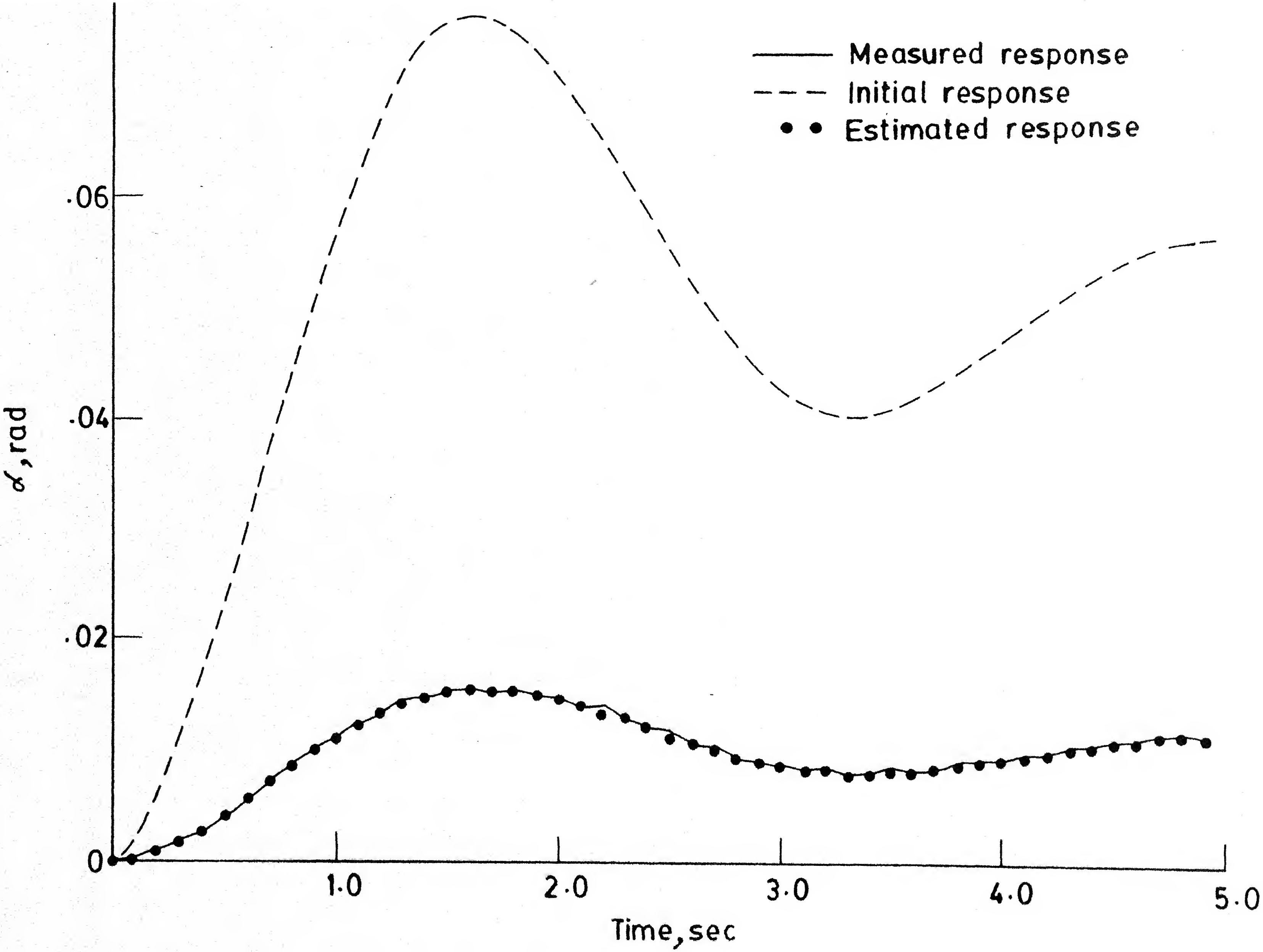


FIG. 7–Continued

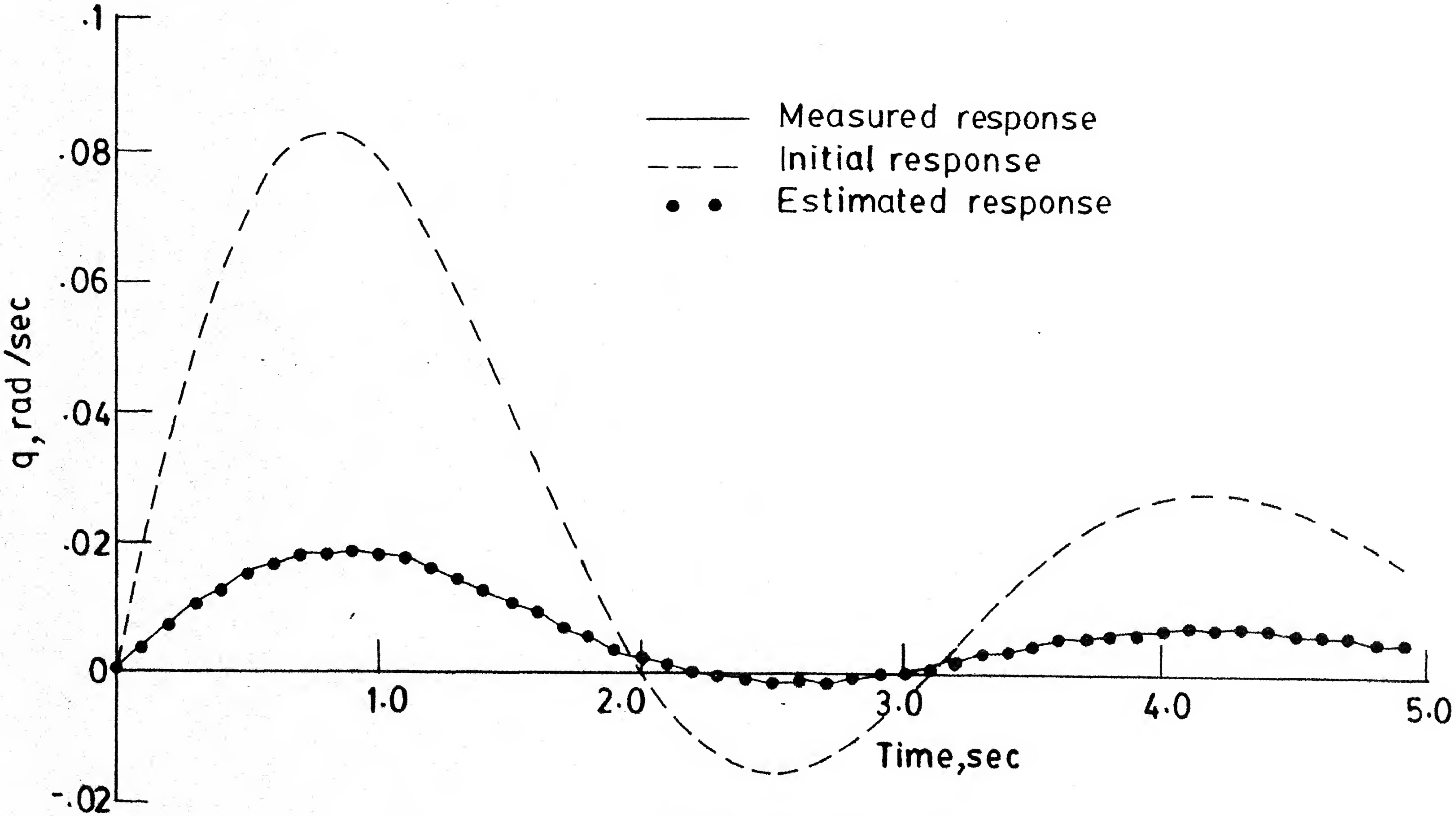


FIG. 7–Continued

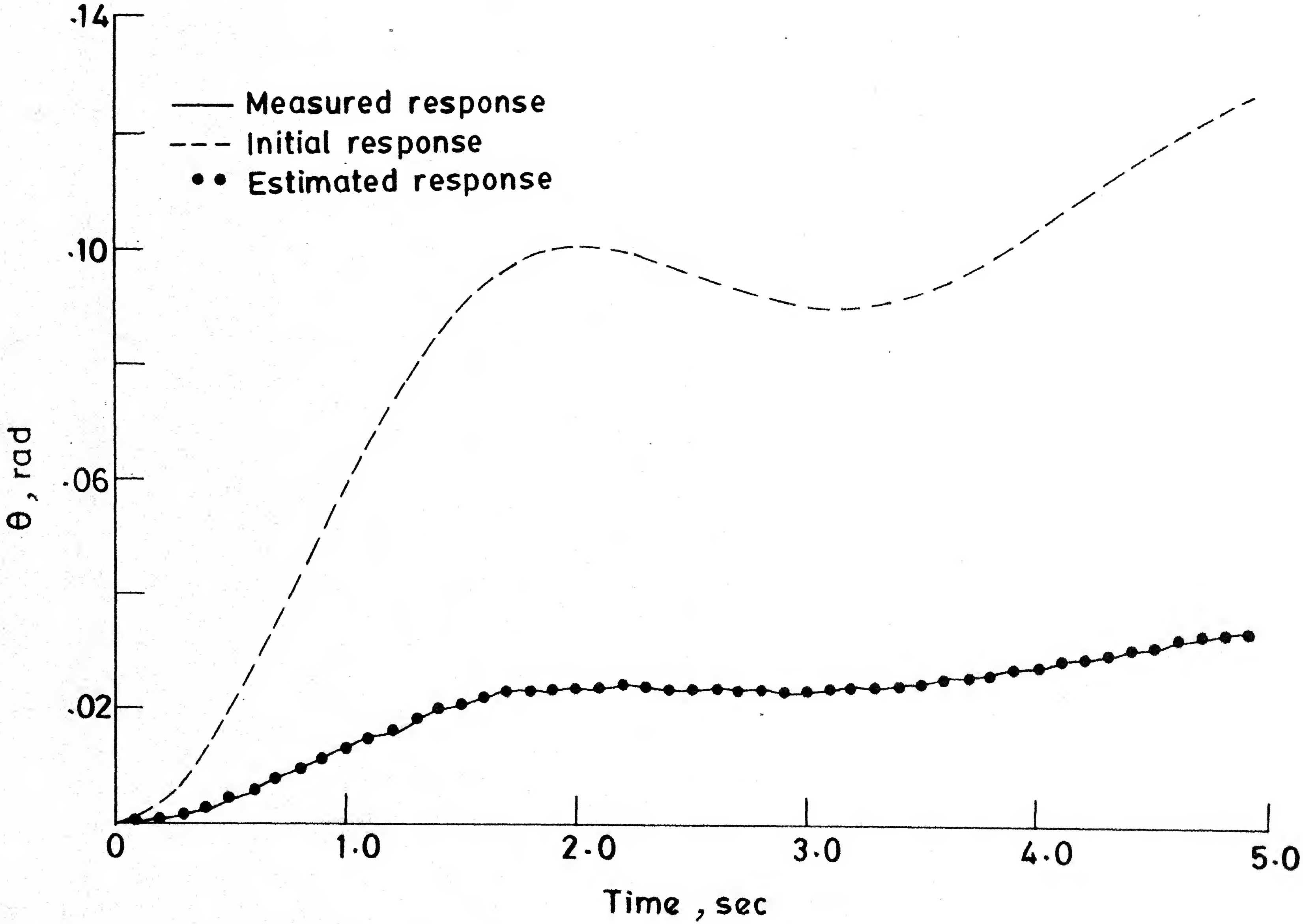


FIG.7-CONCLUDED

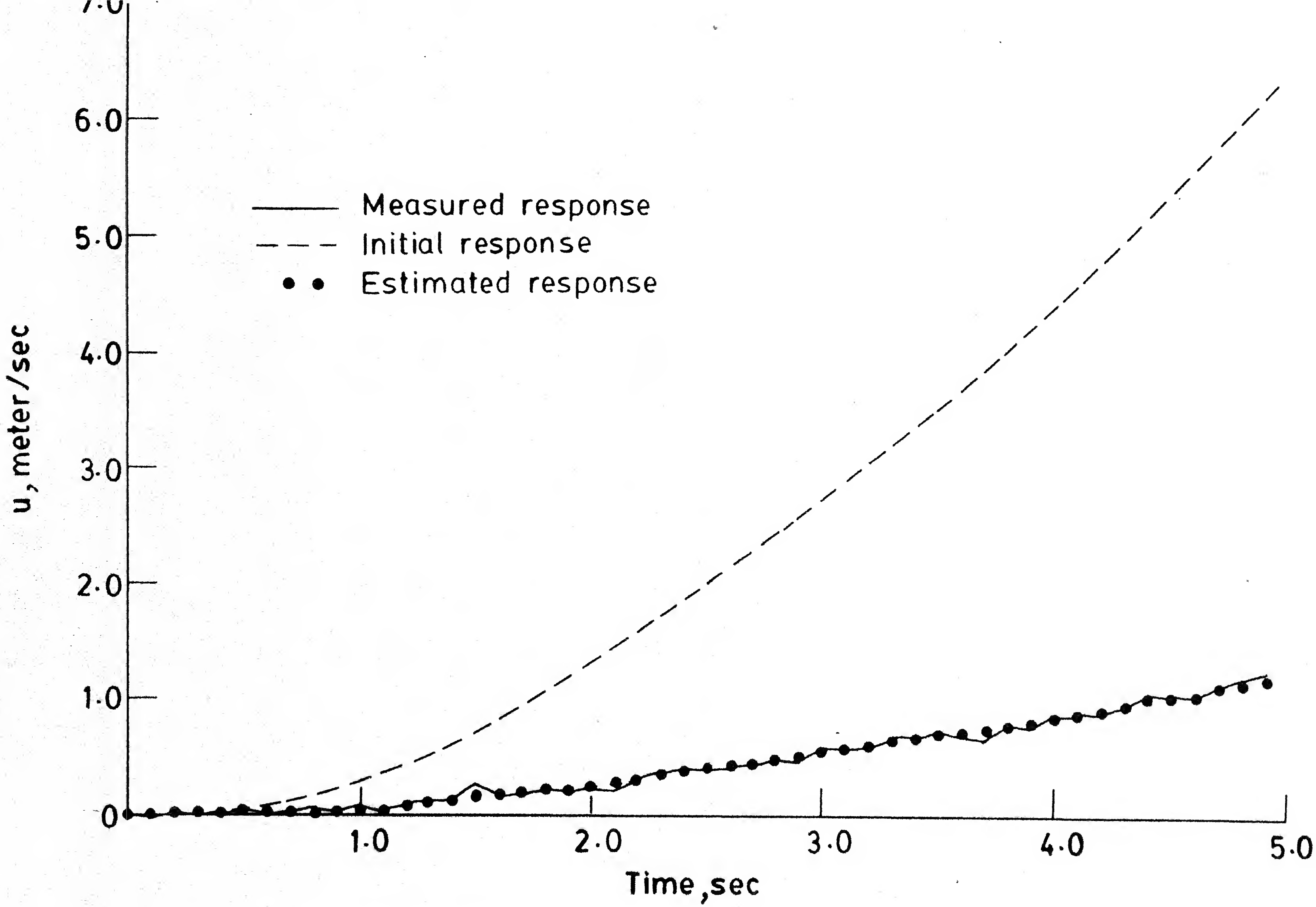


FIG. 8 – COMPARISON OF MEASURED, INITIAL AND ESTIMATED RESPONSE
Location : V2C1 ; Case ; 4; Noise :10 % .

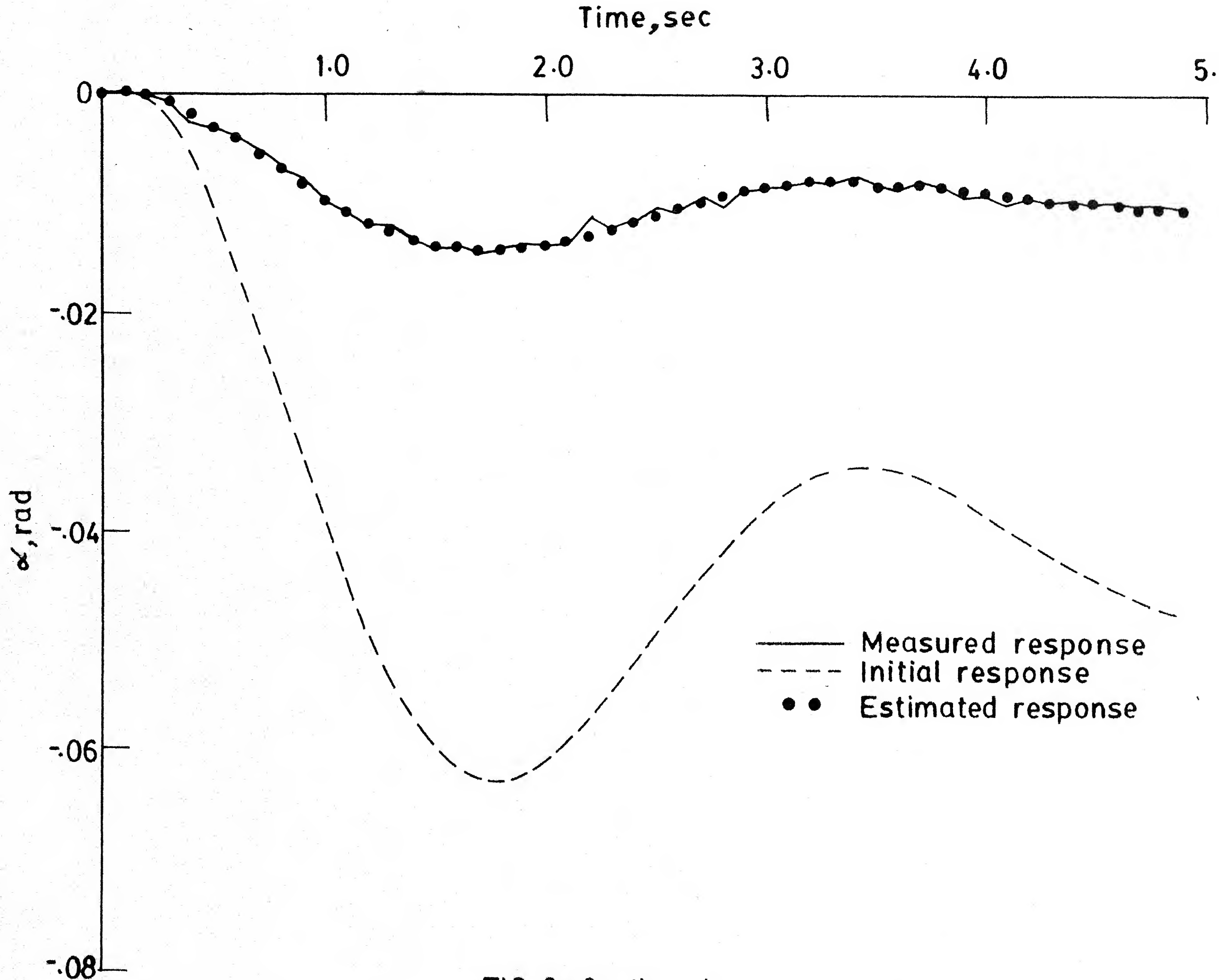


FIG. 8—Continued

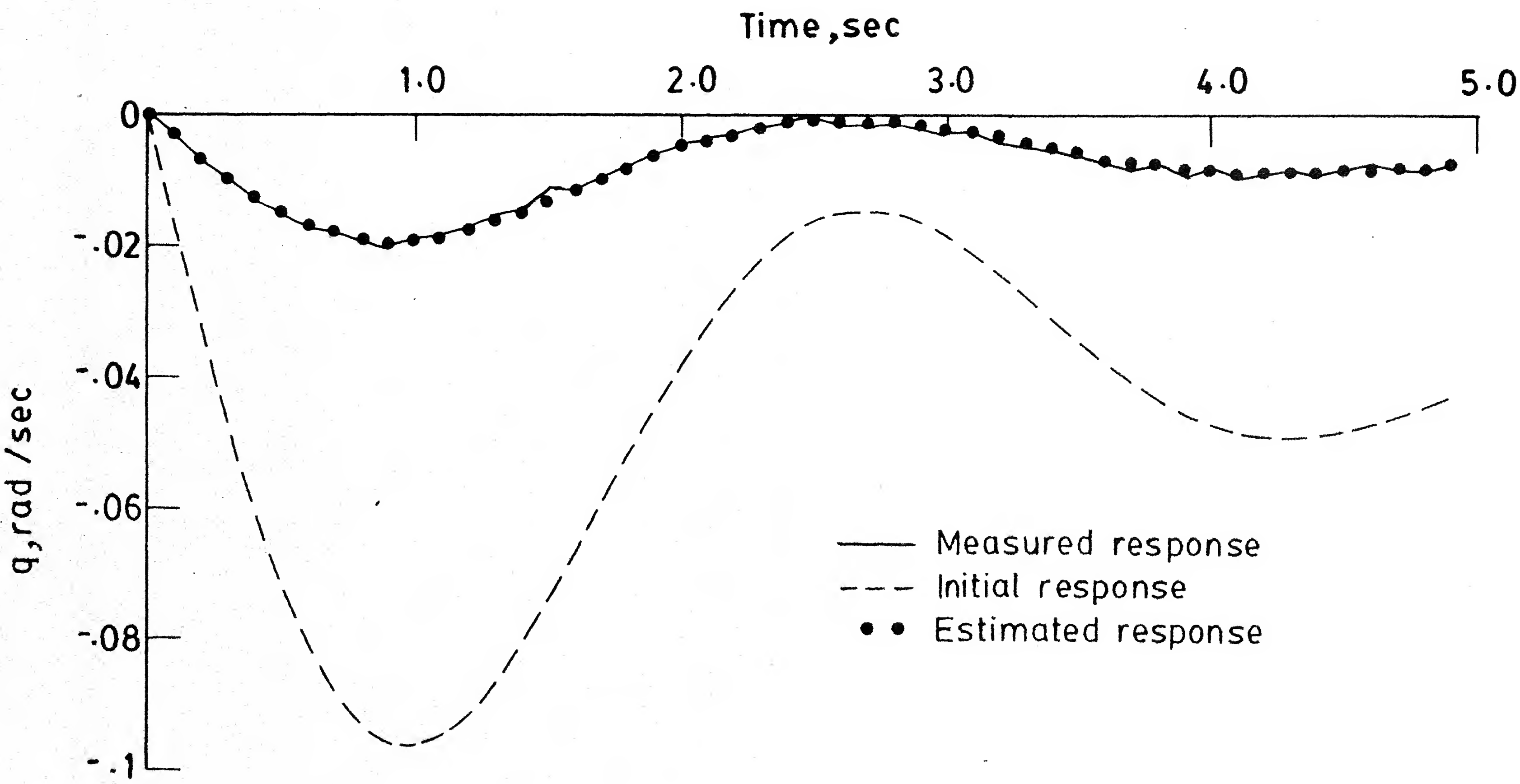


FIG. 8-Continued

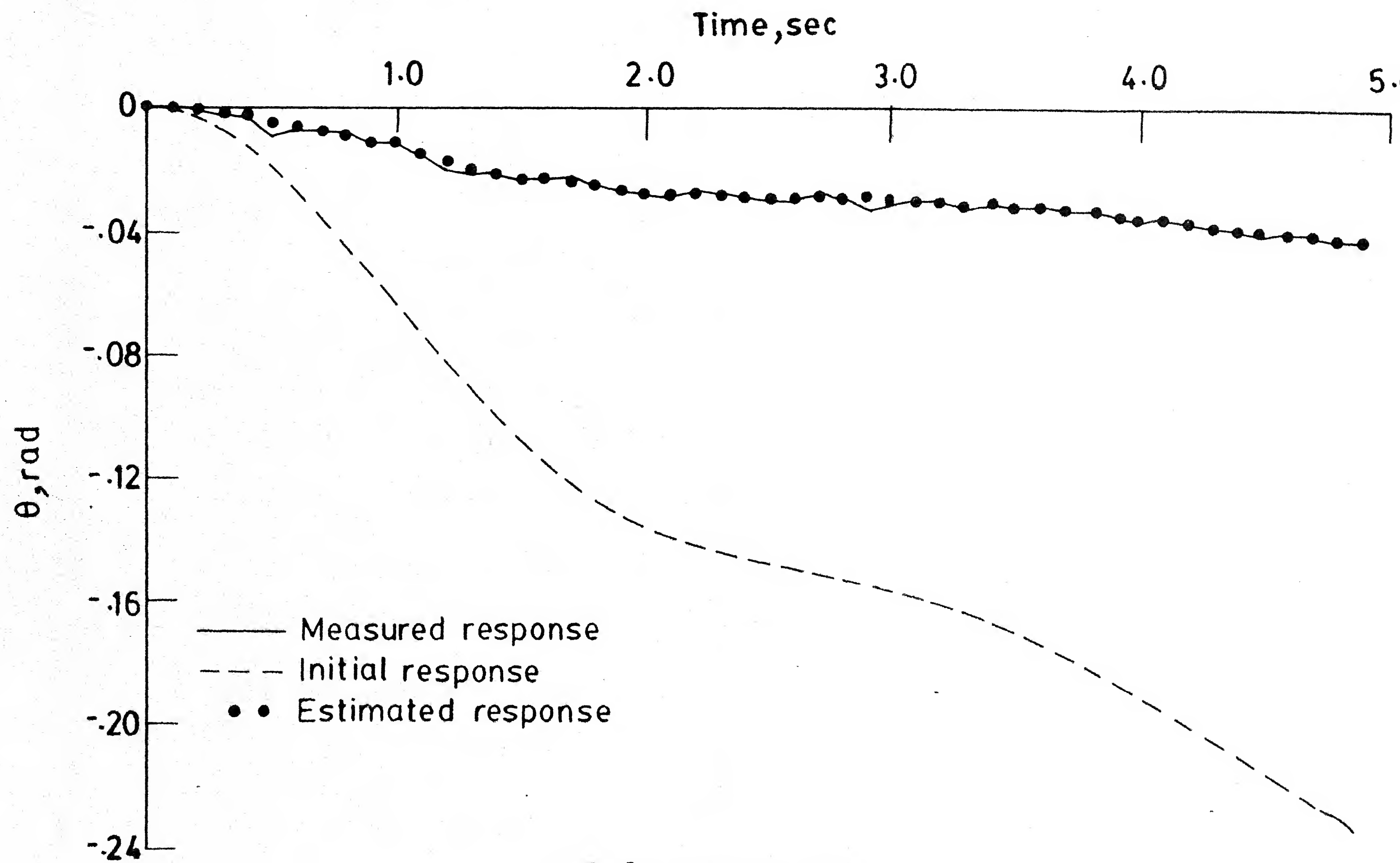


FIG. 8 — CONCLUDED